

**THE STATE OF THE ART  
IN  
CANADIAN MACROECONOMIC MODELLING**

A Report prepared for the

**ECONOMIC FORECASTING DIVISION  
DEPARTMENT OF FINANCE**

By  
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## PREFACE

The Department of Finance employs extensively a large macroeconomic model of the Canadian economy -- the QFS model(Quarterly Forecasting and Simulation Model) -- for economic and fiscal forecasting and simulation exercises. As with any large macroeconomic model, and particularly one that plays a role in the public policy debate, there is an on-going analytical and econometric research programme associated with the QFS model. The research plan underlying the QFS model stresses a commitment to building on current Canadian modelling research rather than pursuing a totally independent model development strategy. The QFS model should be in the mainstream of Canadian macroeconomic models, exhibiting generally accepted economic properties and incorporating current analytical research.

In the course of this research programme we have amassed and reviewed a large body of literature on Canadian macroeconomic models. In particular, we gained valuable insights into the dynamic properties of the various Canadian macroeconomic models from the 1982 Seminar on Responses of Various Models to Selected Policy Shocks, sponsored by the Bank of Canada and the Department of Finance. However, for our purposes, we required an up-to-date and comprehensive source document on the structure and underlying theory in the various Canadian macroeconomic models, and a review of the general themes or trends in Canadian macroeconomic modelling. It was to this end that we commissioned a report on the "state of the art" in Canadian macroeconomic models. The report, which was prepared by Grady Economic & Associates Ltd., provides valuable background material for our modelling efforts, and as well, is an important source document for research studies in macroeconomic modelling in Canada.

I would like to commend Dr. Grady for fulfilling admirably the conditions of our contract to prepare the report; his research was of consistently high quality and, as well, he successfully met our many and varied deadlines when preparing and presenting the many progress reports and drafts of the study. Don Drummond and Heather Robertson of the Economic Forecasting Division were also instrumental in the endeavour: their outline for the study was essentially a "blueprint" of the final product, specifying in detail the format, issues, requisite background information and technical appendices to be completed for the report, and they compiled the extensive reference material that was crucial for the report. Finally, I would like to thank the Canadian modellers -- Ross Preston and P.S. Rao of the Economic Council; Mike McCracken, Carl Sonnen and Elizabeth Ruddick of Informetrica; Leo de Bever and Jan van Vliet of Chase Econometrics; Bill Empey, Tom McCormack and George Vasic of Data Resources Incorporated; Peter Dungan, Joan Head, John Bossons, Mary McGregor and Andre Plourde of the University of Toronto; Ernie Stokes, Anselm London and Peter Gusen of the Conference Board; John Helliwell of the University of British Columbia; and, Jean-Pierre Aubry, Claude Simard, Jack Selody, David Rose and Patrice Muller of the Bank of Canada -- for their generosity in providing background information and comments on the various drafts of the report and for their interest and participation in the Conference on Canadian macroeconomic models that the Department of Finance hosted when the penultimate draft of the report was completed.

As well as being an important step in our own modelling research, this has allowed us to establish networks for future research in Canadian macroeconomic modelling. We would like to extend these contacts in the modelling community, and to expand the exchange of information on Canadian macroeconomic models, both through our own modelling efforts and by maintaining a current body of research material and updates to this report.

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## **1 INTRODUCTION**

The object of this study is to describe the current state of the art in Canadian macroeconomic modelling so as to provide assistance in establishing model development goals for QFS, the Quarterly Forecasting and Simulation Model of the Department of Finance. This study deals only with the structure of models and not their dynamic properties. The results and analysis of the responses to selected policy shocks of versions of many of the models studied here can be found in the report and background papers of the Bank of Canada/Department of Finance Comparative Models Seminar held in July 1982.(1)

The macroeconomic models examined, with the dates of the particular versions studied in parentheses, are:

1. CANDIDE 2.0 (October 1979), Economic Council of Canada;
2. TIM, The Informetrica Model (March 7, 1984), Informetrica Ltd.;
3. RDX2, Research Department Experimental Model, Second Version (1976), Research Department, Bank of Canada;
4. CHASE Econometric Model (April 30, 1984), Chase Econometrics;
5. DRI Model of the Canadian Economy (1983 B, Summer 1983), Data Resources Canada;
6. FOCUS, Forecasting and User Simulation Model (February 13, 1982), Policy and Analysis Programme, Institute for Policy Analysis, University of Toronto;
7. MTFM, Medium-Term Quarterly Forecasting and Simulation Model (September 1984) Conference Board Of Canada;
8. QFS, Quarterly Forecasting and Simulation Model (December 1983), Economic Forecasting Division, Department of Finance;
9. RDXF, Research Department Experimental Forecasting Model (December 1983), Research Department, Bank of Canada;
10. MACE (December 1983), Professor John Helliwell, University of British Columbia; and
11. SAM, Small Annual Model (April 1983), Research Department, Bank of Canada. The references utilized in preparing the paper are listed in Appendix 4. The basic sources on the structure of each model are indicated with an asterix.

CANDIDE, TIM, MACE and SAM are annual models. RDX2, CHASE, DRI, FOCUS, MTFM, QFS, and RDXF are quarterly. For the purposes of analysis, it is convenient to group CANDIDE and TIM together as large annual models. RDX2, CHASE, DRI, FOCUS, MTFM, QFS, and RDXF are grouped as quarterly models with RDX2 preceding the others as it represents the earlier generation model with the largest impact on subsequent model development. MACE and SAM are grouped together as small annual models.

The main areas of investigation of this study are the underlying theory, structure, and steady state properties of the various models. The RDX2 model of the Bank of Canada is utilized as the base model, or the state of the art in macroeconomic modelling in the previous decade. It serves as a reference for comparing current models.

The body of the paper is divided into four parts. The first provides an overview of Canadian macroeconomic models. It describes the size, basic model structure, theory, method of estimation, steady state properties and the use of the eleven models. The second gives a sectoral description of the theory and structure for a grouping of ten broad sectors for the eleven models. These sectors are: 1) Financial sector; 2) Wealth and Permanent Income; 3) Wages and Prices; 4) Aggregate Supply; 5) Income Distribution; 6) Housing; 7) Consumption and Savings Rate; 8) Labour Supply; 9) Trade; and 10) Government. The third part of the paper contains an evaluation of each of the eleven models overall and by sector, based on criteria relating to the current model development goals of QFS. The fourth part offers some observations on the general trends in Canadian macroeconomic modelling and makes some suggestions for future research.

A summary of the key features of the eleven models considered is provided as Appendix 1.

## **2 OVERVIEW OF CANADIAN MACROECONOMIC MODELS**

### **2.1 Philosophy**

The Canadian macroeconomic models considered reflect an evolution of the tradition of econometric model-building going back to Tinbergen's pioneering work at the League of Nations in the 1930s (2) and Lawrence Klein's work starting in the late 1940s (3). The intellectual framework behind this tradition was originally Keynesian with demand determining output and employment levels, but over time it has become eclectic with supply side features and financial variables assuming an added importance. The intellectual evolutions can at least in part be attributed to events such as the emergence of inflation as a serious problem in the late 1960s and the energy price shocks of the 1970s.

It is only in the very short run that the Canadian models studied are demand driven. All of the models have some elements which can be viewed as monetarist. One such element is an accelerationist wage equation. With the exception of SAM the philosophical differences among the models concerning the validity of a Keynesian versus a monetarist approach are more a question of degree and are not fundamental. Of the quarterly and large annual models, FOCUS perhaps puts the most emphasis on steady state and long run properties and market clearing mechanisms.

A noteworthy difference of approach among the models concerns whether or not they incorporate detailed industry output and price sectors. This feature of models takes its inspiration from the input/output tradition of Wassily Leontief. Annual models blending Keynesian macroeconomics and input/output were developed in the U.S. by Wharton EFA. CANDIDE was the first Canadian model in this tradition. TIM is a lineal descendant of the CANDIDE family of models. Some of the quarterly models, namely CHASE, DRI and MTFM, also have an industry dimension, but these models are less disaggregated and are not based on a full-blown input/output structure.

RDX2 can be viewed as the prototype for many of the quarterly models. RDXF, QFS, and CHASE are direct descendents and to a significant degree follow the basic structure of RDX2. DRI FOCUS, and MTFM have individual sectors patterned on RDX2.

MACE and SAM stand out from the other models because of their small size. John Helliwell, the principal architect of MACE and the leading intellectual force behind the Bank of Canada's RDX family of models, felt that macroeconomic models were getting too large and fragmented. Consequently, he set out to develop a core macromodel that was aggregated enough to guarantee both timeliness and economic coherence. The result is the macro block of MACE. Another reason for building MACE was to highlight the role of energy, reflecting the impact of the world oil price shocks of 1973-74 and 1979-80. MACE is thus also a two sector macro-model that provides a fairly complete and consistent analysis of the linkages between energy-producing and energy-using sectors of the economy. The model can be utilized to analyze the short- and long-run consequences of energy supply and demand in considerable detail as well as used for

general macroeconomic analysis.(4)

SAM is a model based on a unique philosophy which draws much of its inspiration from the Lucas critique of macro-models (5). SAM was intended to bridge the gap between large structural models and small reduced form models. Because of its focus on policy simulations (particularly monetary policy), many more theoretical constraints are imposed on this model than on the other models. These include constraints on functional form and cross equation restrictions on coefficients. In order to impose these constraints, the model was kept small so as to facilitate its estimation as a complete system.

Many of the constraints imposed on SAM are necessary to produce desired steady state properties. The model has a steady state growth rate determined by exogenous population growth and technological change. Nominal variables are driven by monetary policy. Cyclical fluctuations around the steady state growth path can be attributed to demand factors, deliberate buffering behaviour, expectational errors, planning errors, and other mistakes.

## **2.2 Use of Models**

### **2.2.1 CANDIDE 2.0**

CANDIDE 2.0 is used by the Economic Council of Canada for medium- and long-term forecasting, generating alternative forecast scenarios, and fiscal and monetary policy analysis. The focus on the medium-term comes from the Economic Council's legislative mandate which enjoins it to assess the medium- and long-term prospects of the economy in relation to its potential. This has been interpreted to preclude an emphasis on short-term forecasting. One of the main vehicles for presenting analysis done with CANDIDE is the Council's Annual Review.

### **2.2.2 TIM**

TIM is utilized for short-, medium-, and long-term forecasting, alternative scenarios, and policy analysis. These are provided on a regular basis to subscribers of Infrometrica's National Forecast Service.

### **2.2.3 RDX2**

RDX2 was designed primarily for simulation and policy analysis. It was regularly used for this purpose by the Bank of Canada and the Department of Finance. The inflexibility of RDX2's software and a number of other technical and data difficulties prevented RDX2 from ever being used for forecasting. It was only after these deficiencies were remedied with the development of RDXF that the RDX family of models became a useful forecasting tool.

### **2.2.4 CHASE**

The CHASE model is employed to produce short- (5 years) and long-term (20 years) forecasts, alternative scenarios, and policy analysis. Short-term forecasts are issued quarterly and long-term forecasts twice a year to subscribers. These forecasts as well as alternative projections and analysis of policy are distributed to Chase clients through its publications and discussed at conferences.

### **2.2.5 DRI**

DRI utilizes its model to prepare short-term quarterly forecasts on a monthly basis, medium-term (10 to 12 year) forecasts on a quarterly basis, and long-term (25 year) forecasts twice a year. These forecasts along with alternative scenarios and policy analysis are presented to its clients in publications and at conferences.

### **2.2.6 FOCUS**

The FOCUS model is used for forecasting, policy and simulation analysis with the emphasis on the medium- and long-term. The results of analysis with FOCUS are presented to subscribers of the Policy and Economic Analysis Program of the Institute for Policy Analysis at the University of Toronto. FOCUS is also the Canadian model used in Project LINK.

### **2.2.7 MTFM**

The MTFM model is the vehicle for the Conference Board's short- and medium-term forecasting exercises. These exercises include the generation of alternative forecast scenarios and some policy analysis. Quarterly forecasts and analysis are provided to associates of the Conference Board.

### **2.2.8 QFS**

QFS is utilized by the Forecasting Division of the Department of Finance to prepare the short- and medium-term forecast which is released in the budget economic paper. It is also utilized quite extensively on a regular basis to examine the implications of alternative forecast scenarios and to explore the likely impact of alternative policies. It thus serves as an essential instrument in the preparation of the budget and the fiscal plan.

### **2.2.9 RDXF**

RDXF is used by the Research Department of the Bank of Canada to produce its regular quarterly short- and medium-term forecast. This process includes the consideration of alternative forecast



scenarios and the analysis of policy options.

### **2.2.10 MACE**

MACE is utilized to analyze the linkages between the energy-using and energy-producing sectors of the economy. Examples of energy developments simulated with MACE include pipelines, oil sands plants, gas exports and alternative energy pricing and revenue sharing regimes.

### **2.2.11 SAM**

SAM is mainly intended for medium- to long-term simulation analysis. Since the model was developed at the Bank of Canada, simulations of monetary policy are of particular interest. The model explicitly incorporates a number of constraints which give the model what are regarded to be desirable long-term properties when subjected to a monetary shock.

## **2.3 Theory and Structure**

### **2.3.1 CANDIDE 2.0**

Final demand in the CANDIDE annual model, can be summarized in terms of seven sectors; consumption, business investment, government, exports, imports, and residential construction. The consumption sector only determines the composition of consumption disaggregated by 40 components. The two savings rate equations determine total consumption residually. Nevertheless, consumption can be characterized as influenced by permanent income, the price of consumption goods, changes in the unemployment rate, other demographic factors and savings incentives.

Investment functions for non-residential construction and machinery and equipment for 44 industrial sectors are modelled based on the Jorgenson specification. The inventory change equations follow the accelerator model.

Government expenditure is disaggregated by level of government and is determined by reaction functions which depend on the level of government revenue, the unemployment rate, and inflation.

Exports are influenced by foreign activity, relative prices, and cyclical variables. Imports depend on domestic activity and its composition and relative prices.

Investment in residential construction is determined by phasing single and multiple housing start activity to constant dollar values. Housing starts depend upon mortgage approvals and the

mortgage rate reflecting the cost and availability of mortgage credit. Income, housing stock and demographics also affect housing demand.

CANDIDE 2.0 has disaggregated equations by industry for investment, man-hours, average weekly hours, employment, wages, sector price and import demand. Real activity at the industry level is an important determinant. The final demand components of the model, numbering approximately 160, are transformed to industrial output using the 1971 input/output structure. An adjustment is applied to correct for systematic bias.

The supply side of the labour market is generated in a detailed demographic sub-model. Participation rates are influenced by the unemployment rate, the after-tax real wage, the dependency ratio, the enrollment rate, per capita real pension benefits and structural shifts in the unemployment insurance program. On the demand side, man-hours or requirements at the sectoral level are derived either from an inverted Cobb-Douglas production function or the first order equilibrium condition of a CES production function. The level of man-hours is affected by present and past values of sector output, the real wage, and capital stock. Average weekly hours are influenced by the after-tax real wage, the unemployment rate and trend factors. Employment in each sector is derived by dividing man-hours by average weekly hours. The aggregate wage is a weighted sum of sectoral wage rates. Changes in sectoral wage rates follow the Phillips curve specification incorporating the trend rate of growth of productivity, inflation expectations and the prime age male unemployment rate.

Wage rates are an important input into the determination of final demand prices using the input/output framework which takes into account productivity, import prices and industry specific user costs.

The financial sector of CANDIDE can be divided into six areas: 1) the portfolio of government debt, 2) the portfolio of selected assets held by the non-financial public, 3) the earning assets of chartered banks, 4) the mortgage market, 5) the term structure of interest rates and the exchange rate and 6) the balance of payments. The 90 day finance company paper rate is modelled as a reaction function of the central bank with foreign interest rates, inflation, bank credit and government debt as arguments. Under a flexible exchange rate regime the exchange rate is determined by expected relative inflation rates, interest rate differentials and short-term capital flows.

### **2.3.2 TIM**

The real sector of the TIM annual model has: 1) a final demand block consisting of consumption, investment, government, exports and imports; 2) an industry output block generating RDP; and 3) a labour market block incorporating demography, labour demand and supply and unemployment. The price side of the model is contained in a final demand price block and an income block which also provides the links between sector expenditures and incomes which comprise the heart of the GNE and GNP identities associated with the national accounts

framework. Interest rates and other financial variables are determined in another block so as to be consistent with the money supply and the level of economic activity. There is also an input/output submodel which integrates the real and price sides of the model and provides a bridge from final demand to industry output and from domestic industry prices and foreign prices to domestic final demand prices.

Personal expenditures are disaggregated into fifty categories of personal consumption. Final demand expenditures of the business sector are broken down into investment in inventories, nonresidential and residential construction, and machinery and equipment which are further disaggregated by industry. Both current and capital expenditures are distinguished in final demand for the government sector. There are five levels of government - federal, provincial, local, hospitals and government pension plans. The foreign sector through exports is a source of final demand and some portion of final demand is satisfied through imports.

Industry output is a key concept integrating expenditure and economic activity. The 73 categories of industry output are utilized in the determination of imports, exports, inventories, business investment, and employment. Industry outputs are used in the determination of factor incomes, which are in turn employed in the determination of current dollar industry output and the corresponding prices of industry activity. The input/output system is also utilized to link final demand expenditure to industry output and to link industry prices, import prices and direct taxes to final demand prices.

The outputs of the labour sector are labour force, employment by industry, and unemployment. The unemployment rate enters many parts of the model as an indicator of the state of the economy. Employment by industry is used in determining the factor income of labour, a key concept in the development of industry prices. Industry specific employment also affects female participation rates. The supply of labour is defined through source population and participation rates. Labour market tightness, real income and demographic variables influence participation rates. Population projections are generated by a demographic sub-model.

GDP by industry is the sum of the factor returns to labour, capital, and unincorporated business income. Factor returns to labour are defined for the 17 industrial sectors at a level of detail consistent with the framework of the national accounts and the labour force survey. The wage bill responds positively to changes in the expected CPI, output per person, and average hours worked. Return to capital is defined for 13 sectors plus some further disaggregation within mining and manufacturing. These equations can be interpreted as a sectoral pricing rule.

The financial sector provides the links between monetary aggregates, financial stocks, government debt and interest rates. Interest rates have the most important impact on the real sector of the model as a result of their utilization in the rental cost and return to capital equations. The change in the exchange rate is related to changes in the current account balance by a simulation rule.

### 2.3.3 RDX2

RDX2 was the Canadian quarterly econometric model that had the greatest impact on subsequent Canadian macroeconomic modelling. It was the direct predecessor to RDXF. It also had a heavy influence on the structure of QFS and the CHASE model. Even the two other models, DRI and MTFM, which are less directly related to RDX2, have been influenced by RDX2. The MACE annual model is also a direct successor of RDX2 in that it has the same principal builder and incorporates some of RDX2's features.

The original version of RDX2 was constructed in the Research Department of the Bank of Canada during the 1969 to 1971 period. The last version of RDX2 was built in 1974 and 1975 and was published in 1976. While there were numerous changes in sectoral detail, the core structure of the model remained essentially the same. Thus, the following summary of the 1976 version of the model can be taken as a general description of the model's structure.

RDX2 had a supply side based on a simple equilibrium growth model based on a Cobb-Douglas production function. This production function was utilized in determining interrelated factor demands for machinery and equipment, non-residential construction and man-hours. It also was used to generate a set of variables capturing imbalances between demand and supply. These variables exerted an impact on wages and prices. Nevertheless, the most important determinant of output remained demand.

Consumption is modelled in RDX2 by a permanent income hypothesis with a role for the market value of private sector wealth. Wage and non-wage income were both incorporated separately in the consumption function.

Residential construction expenditures are tied to mortgage approvals, which in turn are driven by the change in assets of financial institutions and the bond rate.

Investment in machinery and equipment and non-residential construction is determined within the framework of interrelated factor demands by assuming cost minimizing behaviour on the part of entrepreneurs. This yields the desired capital/output ratio for each category of investment given expected labour costs and the expected rental price of capital. Investment is determined according to a forward-looking, flexible-accelerator vintage model, in one of the more complicated sectors ever created in a macro-model.

The major categories of government expenditure are disaggregated by level of government and are modelled as reaction functions.

Exports are driven by foreign activities and relative prices. Export prices are a function of the price of domestic output and the Canadian dollar equivalent of foreign prices. Import equations are based on consumer demand theory. Imports are specified to be a function of the domestic

demand activity, relative prices and capacity utilization. Import prices are the Canadian dollar equivalent of foreign prices.

Employment in mining, manufacturing and other business, the main private sector employment category and the residual factor of production, is explained by a distributed lag on desired labour input derived from the production function. Average hours worked cushion the adjustment between actual and desired employment. The supply of labour is a function of demographics.

Wages are modelled on the theory that in the long-run workers bargain for and obtain a real wage that rises with the trend of productivity. In the short run the real wage diverges from the target due to labour market conditions and unanticipated changes in inflation.

Prices in RDX2 for the main categories of aggregate demand are based on a mark-up on normalized unit labour and capital costs. Import prices, capacity utilization, and the cumulant of unintended inventory changes also are important determinants.

The balance sheet of financial institutions is determined in the financial sector. Liabilities are modelled in a constrained portfolio model covering all major liquid assets held by the non-financial public including government debt. The asset side of the balance sheet is modelled taking into account institutional and market considerations. The key financial variable in the original version of RDX2 was a measure of credit availability based on the difference between the actual ratio of chartered bank earning liquid assets to total assets and its desired level. This variable became less important in later versions of the model.

The key short-term interest rate in the model is the rate on 1 to 3 year term government bonds. It is determined by a central bank reaction function, but the practice in simulation was to set the short-term interest rate so as to attain a targetted level of M1. Long-term interest rates are a function of Canadian short-term rates and long-term U.S. rates. The real supply price of capital is also determined in the financial sector. It is the financial variable with the greatest impact on the real side of the economy. It is related to the long-term interest rate allowing for relative supplies of capital and government debt.

Since the original version of RDX2 was constructed before the exchange rate was unpegged, it incorporated a fixed exchange rate model which only allowed fluctuations within the permitted bounds. Subsequent versions added floating exchange rate models. The exchange rate is treated as the price that clears the balance of payments after short-term capital flows induced by exchange rate expectations have stabilized. The exchange rate is thus estimated as a function of the basic balance, the short-term interest rate differential, domestic liabilities, and foreign assets. Short-term capital flows are implicitly determined.

#### **2.3.4 CHASE**

Expenditures are disaggregated in the CHASE quarterly model into consumption, housing, investment, government, exports and imports. Consumption is modelled in 9 categories, including three durable, three non-durable and two service components, as a function of real discretionary permanent income, population and relative prices. A lagged stock term is included for durables. Discretionary income, which is designed to capture the proportion of income available for consumption is defined as disposable income minus personal transfers to corporations (personal debt) and an inflation premium on net personal wealth.

Residential construction is determined by lagged housing starts. Housing starts per capita in turn are a function of the sales price relative to construction costs reflecting supply considerations and of real effective purchasing power per capita and the mortgage rate reflecting demand factors. The price of housing is derived by inverting the housing demand equation. Relative to the consumer price index it is a function of real discretionary income per capita, the mortgage rate, and the stock of existing housing per capita. Construction costs depend on wages, the export price for lumber and the price for energy.

Non-energy investment in non-residential construction and machinery and equipment is a function of the ratio of the lagged capital stock to business output, capturing the accelerator mechanism. Cash flow variables reflecting the financial constraint are also included. Energy investment is left exogenous.

Inventory change is a function of sales and of the ratio of profits to private business product.

Prices are specified as cost mark-up with homogeneity of degree one. Costs include wages, the price of energy, excise taxes, and the price of imports. Productivity terms based on specific industries are also included.

Employment in the industrial composite depends on the marginal product of labour. Hours worked are sensitive to the business cycle and exhibit a downward trend.

Wages in the industrial composite are determined in an equation that relates the change in wage bill in the industrial composite plus supplementary labour income to the growth in gross private business product and the change in the gap between the actual and natural rate of unemployment.

The key output variable in the CHASE model is real private business output. Trend private business output is given by a Cobb-Douglas production function with actual capital stock, trend hours, trend labour demand and trend total factor productivity. Supply is based on the same production function with actual capital stock, labour and hours and trend factor productivity.

Exports are determined by foreign activity, relative prices and the exchange rate. Imports depend on domestic income, relative prices and domestic capacity utilization. Export prices are mostly determined by foreign prices, but some categories allow some pass-through of domestic prices reflecting the market power of exporters. Import prices depend on foreign prices and the

exchange rate.

The model contains a detailed energy sector incorporating production, consumption and revenues from oil, natural gas, and other energy sources and reflecting the National Energy Program and subsequent agreements.

The financial sector models the balance sheets of banks and other institutions, monetary aggregates, and a variety of interest rates. Canadian short-rates are determined by U.S. short-rates. Long-rates are also determined by U.S. long-rates and allow for any difference in price prospects through a relative inflation premium term. The demand for different monetary aggregates is a function of nominal activity, short-term interest rates and some institutional parameters. The exchange rate is a function of short-term interest differentials, the trade balance, capital flows, and a purchasing power parity term.

The model also includes an industry model analyzing expected demand, production, investment, capital stock and employment for a 33 industry breakdown.

### **2.3.5 DRI**

The DRI model has six blocks: 1) the income-expenditure, employment, potential GNP block; 2) the price-wage block; 3) the industry output block; 4) a financial block; 5) a balance of payments and exchange rate block; and 6) an energy block.

Consumption is broken down into eleven categories including three durable goods, two semi-durables, three non-durables and three service categories. The consumption functions follow the Friedman permanent income hypothesis and thus include permanent and transitory income per person, relative prices and other factors for particular categories.

The key housing starts equation includes a proxy for the "burden" of new home purchases, a lagged housing stock term, proxies reflecting credit availability and rental income and dummy variables for Expo 67 and MURBs. The "burden" variable captures the impact that changes in mortgage rates and new housing costs have on disposable income. Residential construction is a distributed lag on housing starts.

Investment is disaggregated into energy and non-energy, and non-residential construction and machinery and equipment. The non-energy non-residential construction and non-energy machinery and equipment equations can both be characterized as "a cash flow augmented neoclassical stock-adjustment model embodying a replacement investment hypothesis". Both are functions of the difference between the desired and actual levels of the stock of capital, real final sales, a user cost of capital term, and real cash flows. The inventory change equation incorporates both planned and unplanned changes in inventories. Planned inventory investment is determined by

the opening stock of real inventories and the sales of goods according to a stock adjustment formulation. The unplanned change depends on the difference between actual and expected sales.

The foreign sector of the DRI model is disaggregated into 10 categories each for exports and imports of goods and 4 categories of service trade. Exports are specified as functions of foreign demand and export prices relative to U.S. wholesale prices. Imports are determined by Canadian demand and relative prices. The model incorporates a variant of the small country assumption regarding pricing. While import prices are based on foreign prices, export prices are largely determined by domestic cost.

The supply side of the DRI model is based on a Cobb-Douglas production function, yielding potential output from full utilization of the existing capital stock and full employment of the labour force. There is also an energy term in the potential GNP equation.(6) The difference between actual and potential output is an indicator of capacity utilization and is the primary channel through which supply and demand imbalances impact on price.

The labour market sector explains employment of the civilian labour force, the unemployment rate, average hourly earnings in manufacturing, and wages and salaries. Consistent with the aggregate production function, employment is determined by real GNP and output prices relative to the real wage rate. A unique feature of the DRI model is that the unemployment rate itself is based on an Okun's law formulation which means that the labour force (and participation rate) is residually determined. There is also a switch which allows the user to specify the part rate exogenously, and have the unemployment rate defined residually. This is principally used in long-term forecasts.

The model's main wage rate, average hourly earnings in manufacturing, is determined in accordance with an extended Phillips curve specification as a function of inflation and the gap between the actual and full-employment unemployment rates. The price sector of the DRI model also incorporates some novel features for a quarterly model. It is based on a stage-of-processing framework. Unit costs for industries are built up using an input/output price matrix from the costs of labour, capital, and raw materials. These unit costs are used for, wholesale, retail, and national accounts prices.

The financial sector contains interest rates, the money supply, Canadian-dollar assets and liabilities of chartered banks, and deposits of trust and mortgage loan companies. The 90 day paper rate and the bank rate are the key short rates in the model. The 90 day paper rate is determined by the reaction of the Bank of Canada to inflationary expectations, U.S. interest rates and the growth of loans. The longer term rates are modelled from term structure relationships modified to include U.S. interest rates, inflation expectations, and institutional considerations.

The exchange rate formulation is eclectic with interest rate differentials, the basic balance portfolio considerations and expectations playing a role.



The DRI indexes of gross industrial output are based on the profile of final and inter-industry demand using the 1978 input/output structure.

DRI also has an energy accounting sub-model which portrays the current domestic energy pricing and taxation framework.

### **2.3.6 FOCUS**

The documentation describes the FOCUS quarterly model in terms of the familiar IS-LM paradigm. The IS schedule which shows the level of aggregate demand corresponding to a given real interest rate is represented in FOCUS by equations for consumption, investment, government spending, and exports and imports as well as equations for the various components of income. Approximately 55-60 percent of the equations in the model fall into this block.

The consumption sector of the model is based on the permanent income hypothesis with a role for transitory income and relative prices. It also incorporates a detailed model of consumer demand for new and used automobiles with a Jorgenson type user cost and stock effects.

The FOCUS investment sector is novel. It is based on the assumption that private sector output is produced by N nearly identical plants. There is also assumed to be an optimal plant size and within any given plant production is constrained to follow a Cobb-Douglas production function with labour and machinery and equipment as factor inputs. Non-residential construction is viewed as determining the scale of the economy as measured by the number of firms. It is specified as a function of the differential between the internal rate of return on structures and the real after-tax rate of interest on government bonds. Labour and machinery and equipment are treated as partly overhead and partly variable factor inputs within a Cobb-Douglas production framework. They are thus substitutes for each other. The demand for both, however, is complementary to the scale variable, non-residential structures.

Inventory change is based on a partial adjustment model of desired to actual stocks, where desired stocks are a function of expected sales.

The government sector is disaggregated by level of government and expenditures are exogenous in real terms.

Exports are a function of foreign activity levels and relative prices. Export prices are based on both domestic costs and foreign prices. Imports are determined by domestic activity, capacity utilization and relative prices. Behavioural equations tie import prices to U.S. export prices.

The supply of labour is modelled for six age/sex categories. The demand for labour is derived from the same aggregate production function utilized in the investment sector.

The main wage equation in FOCUS is an extended Phillips curve with wage inflation being a function of the gap between the actual and natural unemployment rate and both past and future rates of inflation.

The key price variable in the model is the implicit price index for privately produced GNP. A noteworthy feature of FOCUS is that it contains two alternative price regimes: one based on a mark-up of labour, import, tax and petroleum costs; and another based on a neoclassical market clearing process which determines prices by equating aggregate supply and demand.

Another distinguishing feature of FOCUS is its use of synthetic expectation variables consistently throughout the model. The synthetic expectations series represent expected inflation 90 days, one year, and two years ahead. They were constructed from regressions of actual future rates of price inflation on variables thought to be considered important by market participants in determining inflation such as money supply growth, the unemployment rate, foreign prices, the exchange rate and past inflation.

The LM block of the FOCUS model is more compact than the IS representing some 10-15 percent of the total number of equations. It includes the equations for interest rates and the supplies and demands for various financial assets. The main link between the financial and real sectors is interest rates. The key interest rate is the ex ante real rate of interest on 90 day paper. It is determined by equilibrating the supply and demand for money. The FOCUS model allows for several definitions of money. Other interest rates are generated by estimated term structure relationships.

The exchange rate is also determined by a market clearing mechanism. It is the price that clears the balance of payments.

### **2.3.7 MTFM**

The MTFM quarterly model is characterized by its builders as "based on the neoclassical synthesis". The model is multi-sector. Investment, employment, wages and prices for the sectors are dependent on sector production functions. As with most models, in the short-run output is primarily expenditure determined, but there are supply side feedbacks on prices, imports, exports and thus output from sector capacity constraints.

The MTFM model is structured in fourteen blocks: 1) consumer expenditures; 2) housing and residential construction expenditures; 3) non-residential fixed investment and inventories; 4) government current expenditures; 5) imports; 6) exports; 7) output; 8) employment, hours and labour force; 9) prices; 10) income distribution ;11) taxes and transfers; 12) financial markets; 13) international capital flows and exchange rates; and 14) energy.

Consumer expenditure equations, of which there are 8, are according to the model builders based on the mixed objective-subjective approach discussed by Juster and Wachtel (7). The explanatory

variables include permanent income, transitory income, relative prices, the expected real rate of interest, and consumer confidence as proxied by inflation and unemployment rates. Durable goods expenditure equations are stock adjustment based on investment theory.

The housing sector takes into account housing market stocks and flows and distinguishes between existing and new housing, and single-detached and multiple units. Housing prices and rents are determined mainly in the existing housing market and housing starts in the new housing market. Housing starts of each type are a function of profitability as influenced by housing prices or rents, construction costs and interest rates, and government assisted starts.

Non-residential fixed investment is disaggregated into construction and machinery and equipment for nine industries. Energy investment is treated separately as exogenous. The specifications utilized for the investment equations for the individual categories and industries are neoclassical.

Change in inventories is modelled as a stock-adjustment to the desired stock which in turn is defined as the product of the desired stock-to-sales ratio and expected sales. Discrepancies between actual and expected sales lead to unintended inventory changes. The desired stock-to-sales ratio is affected by the real interest rate.

Government expenditures are disaggregated by level of government and are generally exogenous in real terms.

The export sector exhibits the highest degree of disaggregation for a quarterly model with 15 varieties of merchandise exports and 4 of services. Exports are a function of foreign demand, relative price and a measure of capacity utilization reflecting supply factors. Some export price equations are based primarily on price making behaviour; others on price taking. Imports are also highly disaggregated into 12 categories of merchandise trade and 5 of services. The import equations are based on an import share approach. In the long run the share is based on relative price, capacity utilization and trend factors. In the short run, fluctuations in domestic demand result in a less than unitary elastic change in imports.

Output is modelled for 40 industries in the model using the 1979 input/output tables.

Employment is disaggregated by industry and sex. Industry employment equations specified in person-hours are based on inverted value added production functions and utilize a partial adjustment formulation. In these equations person-hours are determined by industry output, utilized capacity stock and trend labour productivity. Average weekly hours are a function of the percentage change in output and real product wages, trend labour productivity and permanent income. Labour force participation rate equations contain the employment population ratio for both the category and for other categories, as well as other variables such as school enrollment rates by age and sex.

The key wage variables in the model refer to average weekly wages. The wage equations reflect wage-wage emulation and thus include lagged wages as an explanatory variable. They also contain real output, expected consumer prices and the real interest rate.

The price sector is a stage-of-processing approach consisting of three stages: 1) raw material prices; 2) industry prices; and 3) domestic demand deflators and export prices. Industry prices and domestic demand deflators are modelled as a markup on labour, capital, and material costs.

The financial sector contains equations for the financial assets of the public, the balance sheets of the government of Canada and the Bank of Canada, the major assets and liabilities of banks and non-bank financial institutions as well as for interest rates. The key short term rate is the Treasury bill rate. It is determined by a reaction function which has the Bank of Canada responding to the lagged deviation of the money supply from its target, government debt financing, and the exchange rate. Other interest rates result from term structure relationships with some role for new issues.

The exchange rate is modelled as an inverted short term capital flows equation. The important explanatory variables are the basic balance, and the change in the interest differential.

MTFM has a detailed energy sub-model covering oil and natural gas supply and demand, pricing and revenue sharing.

### **2.3.8 QFS**

QFS is a quarterly model which is similar in structure to the RDX series of models developed at the Bank of Canada. Real demand determines output in the short to medium run. Aggregate supply is based on a single Cobb-Douglas production function with four factors, machinery and equipment, structures, labour and energy. This aggregate supply is utilized in employment, investment, and energy demand equations. Imbalances between supply and demand equations have an impact on wages and prices through the labour market gap and inventory disequilibrium variables.

The evolution of QFS is a departure from other models. Its equations were originally developed by sectoral specialists for use in forecasting - a bottom-up approach. The theoretical core was later integrated into the model and sectoral equations were adjusted as appropriate in an effort to pull the equations together and develop a complete model.

Consumption is a function of permanent income, transitory income, and relative prices. Interest rates enter directly into the user cost variable appearing in the automobile demand equation. Housing starts influence durables excluding automobiles.

Residential construction is determined in a housing market model borrowed from RDXF and modified. Housing starts determine supply and the MLS price yields demand. Housing starts

depend on construction costs, the MLS price, and the nominal prime rate. The MLS price is a function of permanent per capita income, a measure of the user cost of home ownership and the lagged stock of housing. The "other" component of residential investment, which is composed mainly of renovations and real estate commissions, is a function of per capita permanent income, the nominal commercial paper rate, and the ratio of the MLS price to the CPI.

Investment in machinery and equipment and non-residential construction both for total and for manufacturing are modelled based on Jorgenson's neoclassical theory. Inventory change follows a partial adjustment to the desired stock and allows for a buffering role for inventories. The desired stock of inventories depends on expected sales and the expected holding cost as measured by the real after-tax rate of return.

Exports of goods are demand functions including terms for real trade weighted indexes of world GNP and a relative price term. Export prices are based on a mark-up on domestic wage and non-wage costs. Imports are also demand equations. Key explanatory variables are permanent income for consumption goods, and investment expenditure for producer categories. The import equations also include a relative price term, the ratio of the respective import price in Canadian dollars and a weighted average of domestic costs as well as a demand pressure variable, the inverse of capacity utilization in manufacturing. Import prices are determined in world markets.

The government sector of QFS is highly disaggregated, particularly on the revenue side. All levels of government are treated separately. Government expenditures on goods and services are exogenous in nominal terms reflecting government budgetary practices.

Employment is derived as a partial adjustment to desired factors of production based on a neoclassical model of profit maximization and the underlying Cobb-Douglas production function.

Average wages per paid employee are determined by an expectations-augmented Phillips curve equation. The labour market tightness term is the gap between the actual and the natural unemployment rate. Price expectations are adaptive.

The price deflators for consumer expenditures are tied to the CPI components. The individual CPI indexes, expressed as growth rates, are determined by a markup on the change in normalized energy, capital, labour and import costs. Investment deflators are linked to the industry selling price index excluding food and beverages, which is also modelled on a cost mark-up specification. Excess demand in product markets influences prices through the inclusion of an inventory disequilibrium term.

In the financial sector Treasury bills and bonds are determined by government financing requirements and the money target as set by the Bank of Canada. Money demand is modelled as a

function of real income, prices, and the 90 day paper rate. In the forecasting version of the model the rate on short-term commercial paper is tied to U.S. short rates, allowing for a partial adjustment to Canada-U.S. inflation differentials. Other interest rates are determined in accordance with term structure relationships.

The exchange rate equations reflect purchasing power parity (as measured by a Canada-U.S. comparison of normalized unit labour costs), the interest rate differential, and the current account balance.

QFS has a detailed energy sub-model reflecting pricing and taxation.

### **2.3.9 RDXF**

RDXF is a quarterly model in the Keynesian tradition. It is largely demand driven in the short-run with comparatively weak supply constraints. A major outlet for demand is provided by imports which put pressure on prices through the exchange rate.

Consumer expenditures are a lagged function of real disposable income (permanent income), relative prices, and financial variables. A unique feature of RDXF which was adopted by CHASE is the exclusion of the inflation premium from the real personal disposable income in the consumer demand equations.

Investment in residential construction is determined by a model involving interaction between builders and homebuyers. Housing starts determine supply and the MLS price yields demand. Housing starts depend on construction costs, the MLS price, and the nominal prime rate. The MLS price is a function of real disposable income per labour force population, construction costs, and the nominal prime rate and mortgage rate.

Non-energy investment in machinery and equipment and non-residential construction is specified according to the neoclassical model with separate terms for the level of demand and the implicit rental cost of capital relative to the wage rate. The investment equations are specified within a framework of interrelated factor demands based on a Cobb-Douglas production function with man-hours, non-energy machinery and equipment, and non-energy non-residential construction as the three factors.

Inventory change is modelled similarly to QFS as a partial adjustment to the desired stock which in turn depends on expected sales and inventory holding costs.

Government spending is broken down by level of government and is exogenous in real terms. Government revenues and transfers are modelled based on institutional detail.

Exports are a function of U.S. and overseas activity and export prices relative to foreign prices.

Supply factors are also taken into account through the inclusion of domestic prices. Export prices generally are specified as a function of the relevant U.S. price converted to Canadian dollars. Imports are determined by domestic activity, relative prices, and a capacity utilization term. Import prices are generally modelled consistent with the small country assumption with U.S. costs converted to Canadian dollars.

The demand for labour (employment) in the private sector is modelled as an adjustment to desired employment as determined by inverting the production function. The labour force is determined by a participation rate equation incorporating a cyclical term, the employment ratio, and a trend.

The growth rate of wages in both the private and public sectors is based on an extended Phillips curve with expected inflation proxied by lagged values of the rate of change of the CPI. The ratio of the output price to the consumer price index is also included as an additional explanatory variable reflecting the demand for labour

Prices are modelled as a flexible mark-up over costs including wages, capital costs and the price of energy, raw materials and traded goods. The mark-up is influenced by capacity utilization, increasing temporarily during periods of excess demand.

Interest rates, the monetary aggregates, and some of the assets and liabilities of various financial institutions are modelled. Documentation on the equations for the monetary aggregates was not provided. Long-term rates are related to short-term rates through the term structure. The nominal long-term rate minus inflationary expectations, defines the real supply price of capital.

The exchange rate responds in the short-run to interest rate differentials, short term liabilities and the basic balance. In the long-run it gravitates to the level dictated by purchasing power parity with the United States.

### **2.3.10 MACE**

MACE is an annual model with has two main industrial sectors, one is the energy producing, and the other uses labour, capital, and energy to produce everything else. The output of the energy using sector is imperfectly substitutable for non-energy imports, but to an equal extent for all final demands. It is also substitutable but to a different degree for foreign output.

The supply side of MACE deals with output and factor demands making use of a vintage CES bundle of energy and capital nested within a Cobb-Douglas function for gross output based on capital-plus-energy and efficiency units of labour. The production function gives the level of output at normal rates of factor utilization. Actual production is set as a utilization rate, depending on current sales, the current ratio of operating costs to output price, and the discrepancy between actual and target inventory stocks. Inventory changes are the residual

between production plus imports and sales. Non-energy imports are also determined in the supply sector.

The absorption sector of MACE models domestic consumption and non-energy exports. The price-wage block includes the determination of output price, the absorption price, annual earnings per employed worker, the price of non-energy exports, and national income identities. National income is derived from output by subtracting net energy imports and net interest dividends abroad.

Equations for the foreign ownership ratio for business, foreign demand for Canadian financial assets, the exchange rate and the balance of payments are contained in an international finance sector. The domestic financial sector includes the demand for money, the market value of domestic business assets, government transfer payments and identities for government liabilities and net private sector wealth.

The energy sector has six main blocks of equations: 1) the demand block; 2) the Arctic pipeline block; 3) the non-frontier natural gas supply sector; 4) the non-frontier crude oil supply sector; 5) the oil sands supply sector; 6) a final block modelling energy transportation and losses and alternative systems for linking energy revenues to equalization payments.

### **2.3.11 SAM**

SAM is an annual model incorporating three goods: energy; commercial non-energy output and a foreign non-energy good. Only the supply of non-energy output is endogenous. It is a function of capital, labour and energy usage. The real price of energy is exogenous. Energy trade is treated as exogenous so as not to destabilize the current account balance. This requires energy output to be endogenously determined in order to balance energy supply and demand. Domestic energy demand is endogenous as a factor of production for the non-energy good. The foreign non-energy good is an imperfect substitute for domestic output. Thus the import and export equations depend on relative prices. In accordance with the small country assumption, the price in foreign currency of imports is exogenous.

Interest rates, bond and equity prices and the exchange rate are determined in asset markets. Government, household, firm and foreign sector claims are disaggregated. Government liabilities are high powered money, domestic bonds, and foreign currency borrowings net of foreign exchange reserves. Domestic claims on firms take the form of equities. The equity market provides a price for these claims to the physical capital stock. Foreign ownership of capital is linked to direct investment flows, which are modelled as responsive to relative equity return rates and rates of return to capital. The foreign sector also has short-term liabilities to the Canadian household sector.

Asset supplies are determined by the sector issuing the instrument. Asset demands of the household sector are estimated as a system including money, government bonds, equities, and net



foreign assets. The supply and demand for assets determine the price of government bonds, the price of equities and the yield on foreign assets. Since foreign interest rates are exogenous, the supply and demand for foreign assets actually determines the expected rate of depreciation/appreciation and the level of the exchange rate. In the long run the exchange rate is determined as part of the general process whereby all market equilibrium conditions are satisfied, and not by any specific rule such as purchasing power parity. But given a fixed real exchange rate, relative purchasing power parity will be satisfied in the long-run. Financial yields and expected yields are formulated to reflect both payment flows and capital value changes. Bond and equity prices are endogenous, and present value functions are specified to relate asset prices to expected future flows and capital gains.

Government expenditure/revenue categories modelled include: transfers to persons and corporations, interest payments, unemployment insurance payments, government wage and non-wage expenditures and personal corporate and miscellaneous taxes. Personal taxes have an important residual finance role, because interest payments on the debt are, at least eventually, funded from personal tax revenues, with the tax rate set accordingly.

Government fiscal and financial variables have an impact on private sector decisions. Firms seek to minimize the cost of producing expected output and adjust their factor usages accordingly. Households plan their consumption and labour supply on the basis of intertemporal utility maximization subject to a lifetime budget constraint. The two financial variables affecting this decision are the initial value of financial wealth and the discount rate. In the short-run, money has an additional direct influence on consumption through a term reflecting the extent to which real money balances exceed their long-run desired levels. Tax variables have an impact on relative factor prices, the return to labour, and the value of human wealth.

Firms, households, and government interact in labour and product markets. SAM incorporates market disequilibrium adjustment equations for output prices and wages. The level of inventories relative to desired levels influences the rate of change of output price and input costs. The unemployment rate relative to the natural rate affects the rate of increase of wages relative to expected inflation plus trend productivity growth. Inventories and unemployment thus have an important role in cushioning shocks to market equilibrium in goods and labour markets.

Long-run equilibrium is achieved in SAM through adjustment processes specified to be consistent with long-run market clearing. A zero-excess-profit, full employment solution is guaranteed through imposition of these constraints in the calculation of an equilibrium wage to which adjustment occurs in the long-run. Nominal values adjust to levels determined to be consistent with real-balance preferences. The conditions of product market equilibrium (ensuring aggregate demand is at potential in the long run) and of asset market equilibrium are provided through long-run endogenous determination of real-interest-rate differentials with the world and the real exchange rate.

### **2.3.12 Observations**

Since RDX2, interrelated factor demands based on an explicit production function have tended to be adopted by quarterly models as a core theoretical framework. The Cobb-Douglas production function is the most prevalent production function. The slowdown in productivity post-1974 has by and large been integrated in models through a modification in exogenous variables or trend factors. MACE is the main exception. It attributes most of the slowdown to unexpected reductions in sales and abnormally low profitability, with a smaller impact from the substitution of labour for energy. Trend productivity growth is assumed to be constant on the basis of econometric tests.

Investment functions on the other hand have tended to become simpler. Forward looking and vintage aspects have not been followed. More straightforward neoclassical investment models have become more the rule.

The upsurge in inflation has led to the adoption of the extended Phillips curve as the preferred specification for wages. The more eclectic bargaining approach of RDX2 has fallen by the wayside.

The increase in inflation and its impact on savings also led RDXF, CHASE and MTFM to exclude the inflation premium from permanent income. This was an attempt to account for the increase in the savings rate in recent years.

Another move away from RDX2 has occurred in the modelling of the government sector. The reaction function approach has fallen from favour. Current quarterly models incorporate specific policy assumptions setting government goods and service spending as exogenous in real or nominal terms.

The floating of the exchange rate has necessitated a flexible exchange rate regime. RDX2 itself was one of the first models to put forward a floating rate model based on an inverted short-term capital flows/stock equation. Other models, except for FOCUS, MACE and SAM, which determined the exchange rate as a market clearing price adopted similar specifications incorporating purchasing power parity and portfolio aspects as well as factors influencing short-term capital flows.

The movement of the Bank of Canada towards a policy approach based on monetary aggregates has led to a greater focus on monetary aggregates in current models.

Another trend since RDX2 has been the incorporation of industrial sectors or stage-of-processing price models. The CANDIDE annual model and its descendant TIM have always had industrial detail, but the addition of industrial disaggregation to CHASE, DRI and MTFM was a departure. It probably reflected the interest of the clients of the commercial forecasting agencies developing these models.

The two energy shocks in the decade of the 1970s and the implementation of the National Energy Program have caused many models to add large energy sectors. The MACE model was even developed primarily to shed light on energy questions.

## 2.4 Size

Table 1 shows the size of the Canadian macroeconomic models as measured by the number of equations and variables. In terms of size the models divide naturally into three groups. The largest are the two annual models CANDIDE 2.0 and TIM. The great size of these models comes from the level of industrial and commodity disaggregation that goes along with their input/output framework. It is interesting to note the extent to which TIM has outgrown its parent model CANDIDE. The additional disaggregation is largely in the foreign trade, trade prices, and private output sectors. Also CANDIDE 2.0 was built before the National Energy Program was launched so that a detailed energy sector was not included in the official documentation provided.

Next in size come the seven quarterly models. In descending order of size they are CHASE, MTFM, QFS, DRI, RDXF, FOCUS, and RDX2. Judging size by numbers of equations CHASE looks larger than it actually is. The equations provided in the documentation included a significant number which either transformed raw data into seasonally adjusted, or were data definitions. The larger of the quarterly models, apart from QFS, namely CHASE, DRI and MTFM all have disaggregation by industry. This increases the number of equations for private output and domestic prices. QFS has relatively large trade, energy, and government, particularly federal government, sectors. It also has a fairly large number of equations for the CPI. The size of these sectors keeps QFS larger than RDXF and its slightly smaller predecessor RDX2.

In general the trend for quarterly models since RDX2 has been toward greater disaggregation. The increase in equations has occurred in most sectors including consumption and investment. The greatest increases, however, have occurred in private output and prices when industrial detail was added. Large energy sectors necessitated by the increasing importance of energy following the two OPEC shocks and the National Energy Program have also contributed to the increase in model size.

The annual models SAM and MACE (excluding its large energy sector numbering 565 equations) are an order of magnitude smaller than the quarterly models. SAM has only 129 equations (122 of which are stochastic) and MACE 48 equations (25 of which are stochastic). The higher level of aggregation in these models is right across the board from consumption and investment to trade. Their government sectors are not split by level of government, and their financial, interest rate and foreign exchange/balance of payments sectors are relatively small.

A question of interest is the degree to which the sectoral detail of the models can be considered "balanced". The concept of "balance" in model construction can be interpreted to mean different things.

One notion is that the level of disaggregation for different concepts should be the same. For example, it could be argued that employment, wage, investment, output, and price should be modelled at the same level of disaggregation. On the other hand, if disaggregation only improves modelling of some of these components, then an aggregate identity can be used to define the variable at the aggregate level.

Another notion of balance emphasized by Mike McCracken at the recent conference on Canadian macroeconomic models sponsored by the Department of Finance is one of causal interaction among blocks of a model. If Block A determines Block B, which determines Block C, then there may be an imbalance if Block A has say, 50 components, Block B, 2 components and Block C, 60 components. Can 60 components be determined by 2 components? The answer depends on the nature of the blocks. If Block A is wage income, Block B is personal disposable income, and Block C is consumption, then there is no imbalance because the conventional specification for consumer demand equations does not require a high level of disaggregation for personal disposable income. On the other hand, if Block C depends on the disaggregated structure of Block A, such as, for example, if Block A is industry wage rates, Block B the total wage bill and Block C industry prices, then valuable information would be lost as a result of the aggregation process in Block B(8).

From another perspective, the question of balance may be even more subjective and its answer may depend on the intended use of the model. For instance, the MACE model may be considered unbalanced from a forecasting point of view because of its large energy sector. On the other hand, from an energy analysis point of view the MACE model can be viewed as balanced. Just looking at the quarterly forecasting models, it is not clear what the appropriate balance among sectors should be. The models with IO sectors can be viewed as unbalanced from one point of view, but such detail is obviously valuable for particular uses and in demand from clients. Similarly, the large size of the government sector in QFS could be considered an indication of imbalance among sectors, but it is clearly appropriate given the interests of the Department of Finance.

## **2.5 Method of Estimation**

The most commonly used method of estimation for the models is ordinary least squares. It is quite common for models to utilize some sort of correction for autocorrelation, to make use of polynomial distributed lags, and to impose *a priori* constraints on coefficients. Models relying mainly on these techniques are CANDIDE, TIM, CHASE, DRI, FOCUS, MTFM, QFS, and RDXF. DRI also utilized some ridge regressions: MTFM used generalized least squares in its import sector; FOCUS tried full information maximum likelihood for the set of factor demands equations; and CANDIDE used a three stage least squares technique with constraint for its debt portfolio model.

The models that incorporate the most sophisticated estimation techniques are the early generation quarterly model, RDX2, and the smaller annual models, SAM and MACE, where estimation is

not as large of a task. RDX2 had at its core a set of interrelated factor demand equations that were estimated using maximum likelihood techniques. It utilized joint least squares for its imports. Its demand for liquid assets sector was estimated within a Brainard-Tobin framework using generalized least squares. There was also a plan to reestimate the model using structurally oriented instrumental variables which was never completed.

MACE was estimated with two stage least squares for most equations. In addition a maximum likelihood grid search technique was employed to estimate the depreciation parameter for the energy-capital factor bundle and to estimate the elasticity of substitution between capital and energy. The main criterion in choosing the parameter was the goodness of fit of the energy demand equation.

SAM employed full information maximum likelihood for the complete system of technology, factor demands and capacity utilization. The same technique was used for asset demands, and a similar technique for the household consumption and labour supply equations. SAM also relied on an iterative Zellner efficient technique in estimating export and import equations.

## **2.6 Steady State**

It is impossible to ascertain the steady state properties of a model without extensive simulation analysis. However, it is possible to gain some understanding of the tendencies that these models might exhibit by an analysis of their structure. For SAM this understanding can be very good because the steady state properties are built into the model. For other models, the understanding is necessarily much more incomplete and tentative.

The concepts of interest are: the natural rate of unemployment; trend productivity; potential output; the structural deficit; the marginal propensity to consume; real interest rates; inflation expectations; and desired factors of production.

The natural rate of unemployment is not defined in CANDIDE and TIM. It is also not defined in MTFM which also takes an industry and age-sex approach to the labour market. The natural rate of unemployment also was not a concept utilized in RDX2 since it only gained acceptance in the early 1970s after the RDX2 wage sector was already specified.

A natural rate is found in all of the quarterly models except for MTFM. The natural rate corresponds to an average rate adjusted for changes in the composition of the labour force and for other developments such as the Unemployment Insurance regime. The natural rates in the FOCUS and DRI models are based on the work of Dungan and Wilson (9). The natural rates in the CHASE, RDXF and QFS models follow a similar approach. The natural rate in SAM is based on that in RDXF. The MACE model also has a natural rate.

Trend productivity is not defined in the CANDIDE and TIM models. In CHASE, DRI, FOCUS, QFS, and RDXF there is a total factor productivity variable. In the DRI model total factor

productivity grows by 1 per cent per year. In FOCUS it shifts downward after 1973. In QFS it grows 1 per cent per year up to 1973 and .2 per cent per year commencing in 1974. The total factor productivity trend in RDXF is 1.9 per cent before 1970 declining to .16 per cent in 1974 and subsequent years. MACE has a labour productivity index for its production which grows at a steady 1.86 per cent per year. SAM has a kink in its productivity growth after 1973, dropping from 2.1 per cent to 1.6 per cent. The rest of the slowdown can be attributed to factor price changes and cyclical factors. RDX2 had a constant labour efficiency factor.

Aggregate potential output is not defined for CANDIDE or TIM. The quarterly models, except for MTFM which has no production function, have all followed RDX2 in having a trend, full employment or potential output based on the natural rate (or in RDX2's case, an average rate) of unemployment. The DRI and FOCUS models have even gone a bit further using capital stock at a peak or natural rate of capacity utilization in the production function in the place of the actual capital stock. The ratio of actual to potential output is used as an aggregate capacity utilization rate in CHASE, QFS, and RDXF as it was in RDX2.

The MACE model does not have a potential output, but one could easily be defined using the natural rate and production function that are in the model. Instead MACE has a concept called vintage-based synthetic supply that is calculated from the production function using actual inputs. The ratio of output to vintage-based synthetic supply is a capacity variable in the model.

The SAM model has steady state output based on steady state labour supply and the output/labour ratio based on an optimizing decision given factor prices. This corresponds to potential output. The SAM model has very strong mechanisms which cause it to gravitate to steady state output in the long-run. None of the models calculate a structural government deficit.

It is difficult to tell the long-run marginal propensity to consume from an inspection of the consumption sectors of the models. The only evidence available is from the simulations done for the Bank of Canada and Department of Finance comparative models seminar in June, 1982. This evidence is not definitive for the models currently being studied for a number of reasons. RDX2 was already defunct. MTFM did not participate. Some of the models such as SAM and RDXF have been significantly modified since the seminar. Other models have been modified to a less significant extent. Also the simulations presented at the seminar were managed to a certain extent to generate what model builders considered reasonable results and thus may not have reflected model properties.

For what it is worth, the long-run (10 year) marginal propensities to consume resulting from a \$1 billion personal income tax cut were associated with an MPC of .773 in CANDIDE, .8 in TIM, .71 in CHASE, .75 in DRI, .84 in FOCUS, and .817 in MACE. The QFS MPC is estimated to be in the .6 to .7 range. For purposes of comparison, the MPC for RDX2 cited in the documentation was .66, rising from .76 if an adjustment were made to endogenize input and paid rent. This is in the same range as those calculated for the other models. In all cases, the MPCs are significantly less than the average propensity to consume and could cause problems in long-run growth

simulations.

Various definitions of real interest rates are utilized in the models in different places. This is discussed in more detail in the sectoral descriptions below. FOCUS is the only quarterly model with consistent definitions of real interest rates, based on its synthetic expected inflation series.

In most of the models inflation expectations are adaptive. In some of the models such as TIM and MACE expectations are determined by distributed lags on prices and can vary from one equation to another. In others such as CHASE, QFS, and RDXF an attempt has been made to follow RDX2 with its PCPICE and calculate a price expectations variable with fixed lag weights to be used where appropriate. QFS even has two such price expectations series. This approach, however, is not always consistently pursued in the models and estimated distributed lags to capture expectations can also quite often be found.

Other more novel approaches to modelling inflation expectations have also been utilized in some of the models. CANDIDE has an inflation expectation variable based on past money growth as well as past inflation (weight .27 on money and .73 on past prices). FOCUS has a synthetic expectations series calculated using reduced form forecasting equations for 90 day, 1 year and 2 year ahead price expectations. The expectations variables are utilized consistently in the model. In SAM expected inflation is a function of current and long-run expected inflation, with long-run expected inflation being dependent on the difference between the expected money supply growth and steady state output growth.

Labour supply in the models is usually determined by the underlying demographics and participation rate equations. In CANDIDE the demographic block is detailed and the participation rate equations by age and sex group are a function of the real after-tax wage, and Unemployment Insurance and pension variables. TIM also has detailed demographics and the real wage influences the part rate. CHASE part rates are based on trends and the percentage change in employment. The DRI model calculates the labour force by adding up employment and unemployment. Unemployment depends on capacity utilization and the percentage of the labour force between 25 and 54. The FOCUS part rate equations depend on the employment rate, the maximum unemployment insurance benefit, real OAS pensions and trend factors. The group part rate equations in MTFM depend on the employment rate for the group and for all other groups. In QFS the part rate is a function of the deviation of employment from trend, a UI dummy, the real wage relative to expectations, and a time trend. The RDXF part rate responds to the employment/population ratio, the natural rate and a time trend. The RDX2 part rate equation included capacity utilization, net immigration relative to population and the population in secondary school as explanatory variables. The MACE part rate is a function of capacity utilization, the ratio of the natural to actual rate of unemployment, the real wage in efficiency units, and an AIB dummy. The labour supply decision in SAM results from an intertemporal labour-leisure optimization decision by households. Labour supply depends on the ratio of per capita real human and non-human wealth to the after-tax return from labour force participation. An adjustment is made for differences between steady state and actual valuations.

Desired factors of production should in theory be derived from a production function. This was the approach of interrelated factor demands pioneered in RDX2. All of the models except TIM, CHASE, and MTFM have production functions. However, only in FOCUS, MACE, and SAM are desired factors of production rigorously derived from the production function. For the annual models with industrial disaggregation such as CANDIDE and TIM or even for the quarterly models such as MTFM with integrated industrial disaggregation, it would be a very difficult task to consistently apply an interrelated factor demand approach. For the other quarterly models, it would not be as hard.

Basically, all of the models but FOCUS (flexible price version) and SAM are primarily demand driven. Output will increase in response to a demand shock. Short-run productivity is the most important source of the output increase and can persist for a very long time. The models based on consistent interrelated factor demand frameworks such as FOCUS (fixed price version), RDX2, MACE and SAM should exhibit a tendency for supply to eventually catch up with demand provided the factor demands are gap-closing. For factor demands to be gap-closing there must be good links between the disequilibrium terms (abnormal inventories and utilization) and pricing and trade behaviour that bring the economy back to equilibrium. For the other models there is no mechanism to guarantee that supply will ever equal demand.

In the flexible price version of FOCUS prices equilibrate supply from the production function with demand so supply will always equal demand. In SAM there are a number of mechanisms that operate to bring the economy into a steady state where supply equals demand and expectations are met.

FOCUS (flexible price version) MACE and SAM are also the only models that are likely to exhibit strong tendencies to bring the economy back to full capacity (or potential) output. Other models besides FOCUS, SAM, and MACE may exhibit tendencies to return to full capacity, but they will be very much weaker. Whether strong or weak tendencies are more characteristic of the actual economy is an empirical question, which must be answered before it is possible to decide which models more accurately describe the way the real world economy operates.

An accelerationist wage-price sector is the most obvious mechanism which might serve to bring the economy back to potential output and the natural rate. For a model to be accelerationist a number of conditions must be met. First, the wage equation must be an extended Phillips curve based on a gap between the actual and natural rate of unemployment with a coefficient of 1 on expected inflation. This condition is met in DRI, and RDXF among the quarterly models.

The second condition is that prices must be homogenous of degree one in costs. In the DRI and QFS quarterly models this constraint is imposed. In other models it is approximately met.

The third condition is that all costs must be endogenous and respond to labour costs. This condition is not met in many quarterly models including CHASE, FOCUS and QFS among the quarterly models because of exogenous food or energy components.



The fourth condition is that the exchange rate must adjust to ensure purchasing power parity so that foreign costs in Canadian dollars will remain in line with domestic costs. Among the quarterly models the CHASE model has a purchasing power parity term in its exchange rate equation. The QFS and RDXF exchange rate equations are also designed with long-run purchasing power parity in mind.

The only quarterly models satisfying all conditions to be accelerationist are QFS and RDXF.

Among annual models the MACE model is reported to have its own model specific natural rate of unemployment and to exhibit accelerationist behaviour. However, it results from a somewhat different process from that sketched out above. MACE has a capacity variable in its wage equation that produces a similar effect to a labour market gap variable. It has homogeneity in costs with the key cost variable specified as the dual to the longer-run production function. It also has no exogenous costs. As an alternative its exchange rate can be set in accordance with purchasing power parity.

The SAM model would exhibit accelerationist behaviour if monetary policy were utilized to lower the unemployment rate below the natural rate. However, it is difficult to see why the model should exhibit such behaviour in response to a fiscal shock.

Of all the models, SAM is the only one with built in steady state conditions which guarantee that the actual unemployment rate equals the natural rate, actual output equals expected output, and that actual inflation equals expected inflation at a non-accelerating rate.

Another steady state property which is of interest is whether or not real wage growth equals trend labour productivity growth. This depends primarily on the role of productivity in the model's wage equations.

CANDIDE and TIM do not have a productivity variable in their wage equation. In CHASE the percentage change in the wage bill for the industrial composite depends on the percentage change in current dollar gross private business product with a coefficient of .5307 and on the change in the gap between the actual and natural rate of unemployment. Such a specification does not ensure that real wages keep up with productivity.

In the DRI model the key average hourly earnings wage variable that enters in the price equation does not have a productivity term in it, but the national accounts average wage equation does. FOCUS also has no productivity term

QFS has a productivity term in its wage equation.

RDXF has a twelve quarter average of the first difference of total factor productivity in its wage equation. Since its coefficient is 1.435 the real wage must rise more rapidly than productivity.

The RDX2 wage change equation has the gap between trend productivity of labour and the real wage as the key long-run explanatory variable. It also includes a short-run productivity measure.

MACE includes the labour productivity index in its wage equation and constrains the long-run coefficient to one. In SAM the wage also tends towards the wage trend based on productivity in the long-run.

### **3 SECTORAL DESCRIPTION (10)**

[Equations are included in some of the sectoral descriptions. To standardize the terminology while remaining within the normal typewriter character set, the RDX2 J operators have been utilized. JW signifies a distributed lag; JiL an i quarter lag; JiS is an i quarter sum; JiA an i quarter average; JiD an i quarter difference; and JiP an i quarter percentage change. The numbers in parentheses above the terms in the equations are included to facilitate reference.]

#### **3.1 Financial Sector**

##### **3.1.1 CANDIDE 2.0**

The CANDIDE 2.0 financial block contains: 1) a debt portfolio model; 2) an asset portfolio model; 3) the mortgage market; 4) a reaction function for the short-term interest rate; and 5) term structure relationships for other interest rates.

The debt portfolio model links the federal government's financial requirements to the change in government deposits and the composition of debt required to finance the requirements. The level of government deposits is a function of government current expenditures and the finance company paper rate. The composition of federal debt depends on the rate of inflation, the change in the stock of treasury bills held by chartered banks, the ratio of Canadian to U.S. short-term interest rates, and the lagged stock of federal debt.

The non-financial public asset portfolio model consists of nine asset demand functions: 1) currency; 2) chartered bank demand deposits; 3) chartered bank savings deposits; 4) chartered bank term deposits; 5) deposits at other financial institutions; 6) deposits at trust or mortgage companies; 7) CSBs; 8) federal securities; and 9) provincial and municipal securities. The asset demand functions include as arguments a vector of interest rates on each asset, price and activity level.

In simulation the demand functions for CSBs and provincial and municipal bonds are replaced by supply equations.

The mortgage market model determines mortgage approvals and the assets of various financial institutions. Mortgage approvals are important determinants of housing starts.

The key interest rate in the model, the 90 day finance company paper rate, is expressed as a reaction function responding to: 1) U.S. interest rates, 2) past inflation, 3) the level of government debt, and 4) the earning assets of chartered banks. Other interest rates are specified as term structure relationships in some cases incorporating the U.S. long-term rate. The long-term rate on government bonds equation also contains the stock of government bonds as an

additional explanatory variable.

The growth of the money supply has an important impact on price expectations in CANDIDE (entering the price expectations with a weight of .27 compared to .73 for lagged prices). This introduces a direct link from money supply growth to inflation. Other important links between the financial and real sectors are through the inclusion of the nominal interest rate on long-term bonds in the calculation of user cost in the investment equations and through the impact of mortgage approvals on housing starts. The impact of interest rates on the exchange rate is another channel of influence.

Long-term capital flows equations generally follow the flow formulation. Direct investment in Canada and abroad from the U.S. and the rest of the world depends on investment requirements in the recipient country and on foreign and domestic sources of funds. U.S. direct investment in Canada is a function of Canadian investment activity, relative capital costs, and U.S. investment requirements relative to the availability of funds.

Portfolio investment in Canada is separated by level of government and corporate portfolio investment is broken into new issues and outstanding bonds. Determinants include Canadian investment requirements, interest rate differentials (sometimes covered), and foreign sources of funds. Portfolio investment abroad by Canadians depends on interest rate differentials, U.S. investment requirements, and Canadian liquidity.

### **3.1.2 TIM**

TIM has equations for various monetary aggregates and their components including term and notice deposits, personal savings deposits, private demand deposits, total private deposits, and M2. These are functions of nominal income and in some cases the 90 day commercial paper rate.

The main interest rate in TIM is the 90 day commercial paper rate. It is tied to the comparable U.S. rate with a coefficient of .876 by a reaction function. Another variable in the reaction function is the change in the ratio of demand deposits to income. It exerts a negative impact on the Canadian short-term rate relative to the U.S.

The long term government bond rate is linked to the key short-term rate by a term structure equation. It also incorporates the U.S. bond rate and federal government financial requirements relative to M2 as additional explanatory variables. The long-term industrial bond rate is in turn linked to the long-term government bond rate with federal government debt relative to M2 exerting a moderating impact on the differential. The conventional mortgage rate depends on the long-term industrial bond rate and the ratio of real residential construction expenditures to non-residential investment.

The government deficit and financial requirements in TIM are calculated from the government budget constraint. Other than in the interest rate equations, the stock of debt is only used in calculating interest on the public debt. There is an integrated asset and debt portfolio model in

TIM.

Money supply does not directly influence prices or other variables in the model. Instead it is interest rates that have the main effect on the real sector. Nominal long-term bond rates enter in the user cost variables in the investment equations and real interest rates on commercial paper and the average mortgage rate influence housing starts.

TIM divides long-term capital flows into direct and portfolio investment. Direct investment by Canadians abroad is a function of current dollar economic activity abroad and direct investment in Canada depends on Canadian activity. Portfolio investment is also influenced by economic activity, but in addition responds to interest differentials and the exchange rate. A rise in the Canadian dollar will discourage investment in Canada and encourage investment abroad.

### **3.1.3 RDX2**

The financial sector of RDX2 includes equations for the liquid wealth of the non-financial public, the assets of financial intermediaries, interest rates, mortgage approvals, and the supply price of capital and wealth.

The demand for liquid assets is based on the Brainard-Tobin portfolio choice framework. The line of causation in this model is from interest rates to asset demands. The eight categories of liquid assets considered are: currency; demand deposits; personal bank deposits; non-personal term and notice deposits; savings deposits in trust and mortgage loan companies; term deposits in trust and mortgage loan companies; CSBs; and net marketable government bonds. The supply of net marketable government bonds is determined by the government budget constraint operating at

both the federal and provincial level. For the federal government treasury bills are defined to be the residual source of finance.

The basic hypothesis is one of lagged adjustment of the ratio of holdings of a particular asset relative to total liquid assets to the desired ratio. This in turn is specified as a function of rates of return, the ratio of nominal income to total assets, and the inverse of the size of the real portfolio. This formulation requires certain consistency constraints which are imposed in the model.

An important feature of RDX2, not picked up in subsequent models, is the buffer-stock role played by money in the system of portfolio demand equations. This is accomplished by including in equations for liquid assets counted as money a term which temporarily allocates to them a portion of any increase in total assets.

The reaction function for the key short-term interest rate, the 1-to-3 year government bond rate, is specified as a distributed lag on the rate of growth of bank loans, and the rate of increase of the CPI. The percentage change in the stock of Government of Canada bonds and treasury bills

is another explanatory variable as is the U.S. treasury bill rate which is entered with a separate coefficient for the fixed and floating exchange rate subperiods.

An RDX2-like reaction function is a feature that has been adopted by later models. If anything, the specifications have tended to become simpler, putting a much higher weight on U.S. interest rates and downplaying other factors. Interestingly, the RDX2 documentation expressed some skepticism about the utility of a reaction function approach given the Bank's switch to targeting on M1 which occurred around the same time as the RDX2 documentation was being prepared.

The equations for interest rates on medium- and long-term government bonds are term structure relationships incorporating inflation, U.S. interest rates, Tobin's  $q$ , and, for the long-term bond rate, the supply of long-term bonds relative to the size of the total portfolio as additional explanatory variables.

The prime rate also follows the rate on short-term government bonds. Deposit rates depend on the prime rate, the short-term bond rate and other deposit rates. The conventional mortgage rate is influenced by demand factors such as mortgage approvals, the number of housing units, and permanent income as well as the long-term bond rate. The 90 day paper rate is a function of covered and uncovered U.S. treasury bill rates, the rate on non-personal term deposits, and the gap between actual and desired chartered bank liquidity. This latter variable was an important financial variable in the original version of RDX2, which had a widespread effect on financial assets and liability equations. In the latest version its impact was much less pervasive, being largely replaced by interest rates.

The supply price of capital is another important variable in RDX2. It enters in the user cost calculations and is explained relative to the long-term bond rate in terms of the ratio of the earnings of real capital to the earnings on the government debt.

The financial sector of RDX2 influences the real sector primarily through interest rates. The effect of interest rates on the exchange rate is a particularly important channel. A more direct impact occurs in the area of housing investment where mortgage approvals determined in a mortgage market submodel are the main variable driving housing starts. Price expectations in RDX2 are adaptive and money growth has no direct impact on wage and price inflation or expectations.

RDX2 has equations for almost all categories of flows between Canada and the United States and between Canada and the rest of the world.

The general form of the equations for new issues is as follows:

$$F/S = a + b(RC - RF) - c(E_{PFXE} - PFX) - d * J1L(L/W)$$

where  $F$  is the capital flow,  $L$  is the corresponding stock,

W is the lenders wealth or portfolio, RC is the Canadian interest rate, RF is the foreign interest rate, and EPFXE - PFX is the expected change in the exchange rate.

### 3.1.4 CHASE

The CHASE financial sector includes equations for major assets and liabilities of banks and other institutions. Chartered bank purchases of government bonds is a function of funds available for investment. Security holdings depend on the interest differential between 10 year provincial bonds and 5 to 10 year federal government bonds. Business loan demand is expressed as a function of investment, profits and the prime rate minus the 90 day rate. Consumer loans respond to consumer spending.

On the liability side, the various monetary aggregates are modelled. The M1 equation (M1CA) is specified as follows:

$$\begin{aligned} \text{LOG}(M1CA/PGNPCA) = & 30.11710 + 1.01459 * \text{LOG}(GNPCA) \\ & - .00443 * R90CA - .00766 * R90CA(-1) \\ & - .00355 * J1P(GNPCA) - .000543 * J1P(PGNPCA) \\ & - .01873 * QTIMECA + (\text{mail strike dummies}) \end{aligned}$$

where PGNPCA is the GNP deflator, GNPCA is real GNP, R90CA is the 90 day paper rate, and QTIMECA is a time trend variable.

The near unitary coefficient on real GNP combined with the specification of money demand in real terms gives an elasticity of 1 with respect to nominal income. The coefficients on interest rates imply a one-for-one inverse relationship between the demand for money and the 90 day paper rate. This low response probably results from the inclusion of the time trend and the percentage change in real and nominal GNP terms.

M2 is calculated by adding personal and non-personal notice deposits to currency and demand deposits. The deposits are explained by interest rates and net wealth.

Government financial requirements are met in the CHASE model through the issuance of treasury bills and the drawdown of cash balances. Government debt is included in net wealth which is an important variable in some of the asset demand equations.

The key interest rate in the CHASE model is that on 3 month treasury bills. It is tied to the U.S. 90 day commercial paper rate by the following reaction function.

$$RTBCA = RCP90 + .76840 + .32937 * (RTBCA(-1) - RCP90)$$

where RTBCA is the 3 month treasury bill rate in Canada,  
and RCP90 is the 90 day commercial paper rate in the U.S.

This equation ensures that the treasury bill rate is equal to the U.S. commercial paper rate plus .77 plus some proportion of the gap calculated using last period's interest rate.

Another key rate is the McLeod, Young, Weir rate on 10 provincial bonds. It is specified in real terms as a function of the U.S. real interest rate on AA utility bonds. Other interest rates are then modelled as term structure relationships. A wide range of interest rates are included.

The money supply does not impact directly on the real side of the economy in the CHASE model through such mechanisms as expectations, nor does money supply growth directly have any real impact on interest rates. However, interest rates have a significant impact on real activity. Nominal and real prime rates affect durable consumption. The nominal mortgage rate is an argument in the housing starts equation. Interest rates or user costs are not included in the investment equations.

The capital account is not modelled in detail in CHASE. Long-term capital flows are defined by the balance of payments identity. Short-term capital flows are exogenous.

### **3.1.5 DRI**

The financial sector of the DRI model covers the major financial assets and liabilities of chartered banks. Total liabilities and all assets except federal government bonds are either modelled explicitly or are exogenous. Federal government bonds are residually determined. Primary and secondary reserve assets are explained in terms of required and excess secondary reserves. Equations for business and personal loans and holdings of the securities are included. The equations are specified in relation to total uncommitted assets. Personal loans depend on the change in consumption scaled by GNP and on bank liquidity. Business loans are a function of the prime business loan rate and the cash requirements of businesses as represented by investment and bank liquidity. Bank holdings of other Canadian securities are expressed as a function of the differential between an average of corporate, municipal and provincial bonds and the long-term federal government bond rate.

Bank deposits held by the public are demand functions incorporating interest rates, prices, income and GNP components as explanatory variables. Demand deposits, the key component of M1, depend on real GNP, the GNP deflator, the 90 day deposit rate, and a dummy variable for postal



strikes. The elasticity with respect to the GNP deflator is .65 and real GNE .98. The elasticity on the deposit rate at -.05 is implausibly low, as is that on the GNP deflator.

Personal savings deposits depend on the rate on noncheckable savings accounts, the treasury bill rate, disposable income and a dummy variable for postal strikes. Non-personal term and notice deposits are a function of the 90 day bank deposit rate, lagged liquid assets held by the public, disposable income, retained earnings, and the postal strike dummy.

The government budget constraint is not explicitly incorporated in the DRI model. However, deficits are cumulated and the cumulative stock is used as an explanatory variable in the equations for public debt charges and the long-term government bond rate.

The key interest rate in the model is the rate on 90 day finance company paper. It is determined by a reaction function incorporating both the covered and uncovered U.S. commercial paper rate, inflation expectations, and the growth of general loans. Other rates are specified as term structure relationships often incorporating comparable U.S. rates and other financial variables. The key domestic rate in term structure equations for government bonds is the rate on one-to-three year government bonds. Inflation expectations are included in the equation for the longer term government bonds. An important rate because of its impact on investment is the rate on McLeod, Young, Weir industrial bonds. It is a function of the one-to-three year government bond rate, the U.S. rate on new corporate bonds, and inflationary expectations.

Monetary policy in the DRI model impacts indirectly on real activity and prices through interest rates and the exchange rate in the conventional manner. The nominal yield on McLeod, Young, Weir industrial bonds enters the user cost of investment. Nominal mortgage rates affect residential construction through a term measuring the burden of home-ownership. Nominal short-term interest rates have an influence on new car sales through a usercost term.

The DRI model has a number of equations explaining international capital flows. Endogenous components include new issues and retirements of corporate bonds, new issues and retirements of provincial and local government bonds, Canadian direct investment abroad, and foreign direct investment in Canada. Other categories are exogenous.

New issues of corporate bonds respond to cash requirements (investment minus before-tax profits), long-term corporate interest rate differentials relative to the U.S., the stock of corporate bonds, and several dummy variables. New issues of provincial and local bonds reflect the refinancing of retiring bonds, provincial and local deficits, the Canadian-U.S. differential on government bonds, the differential in the U.S. between U.S. local and corporate bonds, and a number of specific events. Foreign direct investment in Canada is a function of after-tax corporate profits, and unit labour costs in manufacturing. Canadian direct investment abroad responds positively to U.S. GNP and the user cost of construction in Canada and negatively to the U.S. user cost of capital.

### 3.1.6 FOCUS

The asset and liability side of the FOCUS financial sector is not very disaggregated. Complete balance sheets for chartered banks and other financial institutions are not specified. The main financial variables modelled are the monetary aggregates, M1 and M2, and interest rates. Total assets of banks, life insurance companies, and trust and mortgage loan companies are modelled in the housing and mortgage sector to generate mortgage approvals, the main determinant of housing starts.

A unique feature of FOCUS is the menu of options facing the user for determining money demand, interest rates and the exchange rate. They are: 1) setting a target for M1 and allowing the 90 day paper rate to equilibrate money demand and supply; 2) setting a target for M2; 3) setting a target growth of chartered bank reserves; 4) setting a target for short-term nominal interest rates; 5) setting a target for the short-term real rate of interest; 6) setting the short rate to meet a target for the foreign exchange rate; and 7) setting a target for real M1.

The equation for M1 (MONEY1NS) which was estimated with the 90 day paper rate as the left hand side (LHS) variable is:

$$\begin{aligned} \text{LOG}(\text{MONEY1NS}) &= .08413 + (\text{seasonal dummies}) \\ &+ .0667 * \text{DUMTAXREFORM} \\ &+ .8265 * \text{LOG}(\text{GNP}) - .0374 * \text{RMFINCO90NS} \end{aligned}$$

where DUMTAXREFORM is a dummy for the 1972 tax reform, GNP is nominal GNP, and RMFINCO90NS is the 90 day finance company paper rate.

The equation for M2 has the same structure.

There is no explicit government budget constraint in FOCUS. Government debt is not specified.

The main short-term interest rate in FOCUS is the 90 day paper rate. It is determined according to one of the options noted above. The real value then depends on the expected rate of inflation.

A novel feature of FOCUS is its use of synthetic expectations variables for price expectations, 90 days, 1 year, and 2 years in the future calculated utilizing reduced form forecasting equations for prices. This model for expected inflation 90 days in the future includes such variables as past domestic inflation, past U.S. inflation, the rate of growth of M1, real GNP, the price of foreign exchange and the deviation of the unemployment rate from its two year average. The specification for inflation 2 years in the future is simpler depending only on consumer price inflation and the deviation of money supply growth from its two year average. Expected inflation one year in the future is based on 90 day and 2 year expectations with the 90 day expectations

having almost all the weight.

The rate which is key for the real side of the economy is the real 1-to-3 year government bond rate. It is a function of the real yield on 90 day paper. The nominal 1-to-3 year government bond rate is calculated by adding inflationary expectations to the real rate. This rate plus the rate of change in consumer prices provides the basis for the term structure equations explaining the longer term government and industrial bond rates.

The prime rate depends on the bank rate which is exogenous. It is clearly not satisfactory to have key rates such as the bank rate exogenous. Thus, in a more recent version of the model, short-term interest rates are endogenized by inverting the money demand equation. The mortgage rate is a function of the long-term industrial bond rate and mortgage approvals relative to housing starts.

FOCUS has a number of channels by which financial variables affect real quantities and prices. The impact on price expectations and hence wages and prices has been mentioned as has the effect of mortgage approvals on housing. The other main channel is through interest rates. Capital flows in FOCUS are more sensitive than in the other quarterly models to changes in interest differentials and with a market clearing exchange rate the effect on trade flows and domestic prices can be substantial. The real rate on 1-to-3 year government bonds is an important component of the user cost variable employed in the investment equations. The 90 day paper rate is an argument in the user cost of automobiles and thus has an influence on consumption.

FOCUS contains several equations for long-term direct and portfolio capital flows based on the portfolio approach and an aggregate equation for short-term capital. Equations for the stocks of direct investment holdings, and long-term liabilities outstanding are also included.

Foreign direct investment in Canada is explained in terms of financing pressure in Canada, the gap between the after-tax rate of return in Canada and the U.S., nominal imports, and the stock of direct investment. The stock term exerts a depressing impact on direct investment. The equation for Canadian direct investment abroad is broadly similar.

The main categories of long-term capital portfolio investment are net new issues of provincial and municipal, and corporate bonds. Net new issues of provincial and municipal bonds are a function of the deficit of provinces and municipalities adjusted for the availability of C/QPP funds, the interest differential between Canadian provincial and municipal bonds and U.S. state and local bonds, and the stock of Canadian provincial and municipal bonds outstanding relative to the stock of U.S. state and local. Net new issues of corporate bonds are determined by a distributed lag on the Canada-U.S. interest differential on industrial bonds scaled by GNP. Trade in outstanding bonds and stocks depends on the relative growth rates of Canadian and U.S. nominal GNP, the differential between Canada and the U.S. on 3-5 year bond yields, and the change in the short-term differential.

### 3.1.7 MTFM

The MTFM financial sector has equations explaining the financial assets of the public, the balance sheets of the government, the Bank of Canada, the major assets and liabilities of the chartered banks and non-bank financial intermediaries, and a number of interest rates. It also has a federal budget constraint with financial requirements being financed through the issuance of long-term bonds. Other financing mixes are also possible.

On the asset side of chartered banks' balance sheet, personal loans depend on nominal consumption and the bank rate. Business loans are a function of investment, the gap between the prime rate and the corporate paper rate, and the gap between the prime rate and the U.S. industrial bond rate. Mortgage loans respond to the mortgage rate and income. Demand for provincial and municipal government securities are modelled separately depending on the gap between the interest rate on provincial-municipal securities and the treasury bill rate.

Monetary aggregates are also modelled in MTFM.

The money demand equation (M1) is:

$$\text{LOG}(\text{FM1}/\text{PGNP}) = -.00395 + \text{J3W}(\text{LOG}(\text{YGNPK})) + \text{J3W}(\text{RCP90})$$

where FM1 is M1, PGNP is the GNP deflator, YGNPK is real GNP, and RCP90 is the rate on 90 day commercial paper.

The long-run elasticity of real demand for M1 with respect to real GNP is .94132. It is achieved after 3 quarters. The long-run semi-elasticity with respect to changes in the 90 day commercial paper rate is .01689. This indicates that after 3 quarters a 1 percentage point increase in the short-term interest rate would reduce real money demand by 1.7 per cent. This is much higher than in the CHASE and DRI models and appears to be more plausible, but it too could be downward biased by the downward shifts in money demand resulting from the introduction of cash management and daily interest chequing and savings.

Interest rates in the model are determined by a reaction function for short-term rate and term structure relationships.

The Bank of Canada reaction function for short-term interest rates as measured by the 90 day treasury bill rate (RTB90) is:

$$\begin{aligned} \text{RTB90} = & .11470 + .68304 * \text{J1D}(\text{USRTB90}) \\ & - 23.5928 * \text{J1D}(\text{GBALF}(-4)/\text{YGNP}(-4)) \\ & + .38336 * \text{DTARGET1} + .38336 * \text{DTARGET2} + \text{RTB90}(-1) \end{aligned}$$

where USRTB90 is the U.S. treasury bill rate, GBALF is the federal government budget balance, YGNP is nominal gross national product, DTARGET1 is the deviation of M1 from a 2 per cent target band on the upside, and DTARGET2 is the deviation on the downside.

The important arguments in the reaction function are the U.S. treasury bill rate, the change in the federal government budget balance as a percentage of GNP, and the deviation of money growth from target. The term structure relationships explaining other interest rates such as the corporate bond rate, the provincial and municipal bond rates, and the long-term federal bond rate make some allowance for the effects of new issues.

The main impact of the financial sector on the real sector of MTFM is through the traditional interest rate channels. Real interest rates have a direct negative impact on consumer expenditures, housing investment, business investment and wages. They have a direct positive effect on producer and final demand prices as a measure of the cost of capital. Interest rates also have a direct positive impact on interest income. The exchange rate responds directly to changes in interest rate differentials as do international capital flows.

Interest rates affect consumer expenditures via a real interest rate variable. This variable is defined as the industrial bond rate minus the expected rate of inflation in the consumer price deflator (currently an 8 quarter moving average of past inflation). In the case of housing investment, a user cost measure including the mortgage rate, industrial bond rate and expected housing price inflation directly affects housing prices and thus single starts from the demand side, while the prime rate and house price inflation affect such starts from the supply side. The user cost of housing affects rents and thus multiple starts from the demand side, while a user cost of renting for landlords, which incorporates the mortgage rate, impacts on the supply side.

Business investment in plant and equipment is affected by user cost of capital variables. These variables are constructed using a weighted average of the industrial bond rate and the TSE 300 earnings/price ratio, and the expected rate of consumer price inflation, as well as many other tax variables. Inventory investment responds to the differences between the interest rate on 90 day commercial paper and the expected inflation in the price of such goods (again a moving average of past inflation).

Most main components of international capital flows except for net new bond issues of the Government of Canada and its agencies, and net flows of Canadian stocks and foreign securities are modelled. Net new issues of provincial and municipal bonds expressed as a proportion of financing requirements are a distributed lag function of changes in interest differentials. Net new issues of corporate bonds as a proportion of investment spending also depend on a distributed lag on changes in interest rate differentials and on the ratio of retained earnings to investment requirements. Direct investment in Canada is a function of U.S. activity and Canadian before-tax profits.

### 3.1.8 QFS

The financial sector of QFS is not expansive. The financial assets modelled are the monetary aggregates M1, M1A, and M2. The equations for M1 and M1A are specified as demand functions based on the rate on short-term commercial paper, real GNE and the GNE deflator. Polynomial distributed lags are utilized to reflect lagged adjustment. The stocks of bonds, treasury bills, and CSBs are endogenized using a simulation rule based on the government budget constraint.

The equation for M1 (DM1Y) in QFS is as follows:

$$\begin{aligned} \text{LOG}(\text{DM1Y}) = & a_0 + (\text{dummies for special factors}) \\ & + \text{LOG}(\text{PGNE}) + \text{JW}(\text{LOG}(\text{GNE71}) + \text{JW}(\text{RFCP90}) \end{aligned}$$

where DM1Y is M1, PGNE is the GNE deflator, GNE71 is GNE in 1971 dollars, and RFCP90 is the 90 day commercial paper rate.

The long-run elasticity with respect to changes in real GNE is reached after 3 quarters and is .76. The long-run semi elasticity with respect to the change in the 90 day commercial paper rate is .021. This indicates that a one percentage point increase in the interest rate would reduce money demand by roughly 2 per cent. This effect is a little more than in the MTFM model.

A similar equation is in the model for M1A. The remaining monetary aggregate in QFS is M2. In addition to variables for income and the commercial paper rate, the M2 equation includes variables for the rate of return on savings deposits, the rate of return on competing assets, and the stock of CSBs. The specification is partial adjustment.

The key interest rate in QFS is the rate on 90 day commercial paper. It can be determined using a variety of simulation rules. One relates it to the U.S. commercial paper rate plus an allowance for the Canada-U.S. inflation differential. The rule is allowed to function freely between specified real interest rate bands (defined using the year-over-year increase in the GNE deflator). Other rules which can also be utilized relate the 90 day commercial paper rate to the growth of monetary aggregates, the exchange rate, or directly to the corresponding U.S. interest rate.

Other interest rates are tied to the rate on commercial paper through term structure relationships incorporating covered and uncovered U.S. interest rates as additional explanatory variables. The financial sector in QFS affects the real sector through interest rates. It has no direct impact on price expectations. The important financial-real links occur through the user cost variable incorporating the nominal industrial bond yield in the investment equations, a real after-tax return in the inventory demand equation, the nominal prime rate in the housing starts equation, and the user cost in the automobile demand equation. Interest rates also affect the real sector through their impact on the exchange rate and trade flows. A link that has an important but

somewhat unexpected effect is the positive impact of interest rates on non-wage disposable income. The partial effect of this influence is to cause consumption to respond positively to interest rate changes.

QFS does not have any equations for international capital flows. However, there are some simulation rules for long-term capital flows and foreign bonds outstanding which are utilized to improve the dynamics of the balance of payments and to make stock-flow relationships in the model consistent.

### **3.1.9 RDXF**

It is not possible to describe fully the RDXF financial sector as several key equations were not included in the documentation provided by the Bank. The only assets and liabilities for which equations were provided are chartered bank personal loans, savings deposits, and net foreign assets.

RDXF incorporates the government budget constraint. Equations are included to distribute federal financial requirements amongst CSBs, treasury bills, and long bonds. A budget constraint equation for the provincial government is also provided.

The documentation provided had equations for 10 different interest rates including the bank rate, and the rate on GICs, federal government bonds, long-term industrial bonds, personal savings deposits, personal fixed term deposits, and the conventional mortgage rate.

The equations were specified as very complicated term structure relationships incorporating U.S. rates, other Canadian rates and in many cases very long lags. The equation for the key 90 day commercial paper rate that is at the base of the term structure was not provided.

Feedbacks from the financial sector to the real sector in RDXF are judged by Bank staff to be weak. This does not necessarily mean that they are weak relative to other models. The chief linkages are through movements in interest rates, the most important of which is the effect of changes in Canada-U.S. interest differentials on the exchange rate and trade flows.

The main linking variable between the financial and real sector is the real supply price of capital (RHOR) which is a determinant of the implicit rental prices of capital in the investment sector. It is determined by the following equation:

$$\begin{aligned} J1D(RHOR) = & -.01920 + .50985 * J1D(RLIND) \\ & + .00787 * J1D(PCPICE) \\ & - .00817 * (J1P(LGB) - J1D(PCPICE)) \end{aligned}$$

where RLIND is the rate on McLeod, Young, Weir industrial bonds, PCPICE is expected inflation, and LGB is federal and PLH bonds outstanding.

The real supply price of capital thus responds positively to increases in the industrial bond rate and to inflationary expectations (as measured adaptively by a long lag on past percentage changes in the CPI). It is negatively related to changes in the real stock of government bonds, presumably indicating that government bonds are a better substitute for money than for capital.

Other interest rate related links between the financial and real sectors are the inclusion of the prime rate in the housing starts equation in nominal form, in the after-tax holding cost for inventories in real form (prime rate minus the percentage change in the price of goods times one minus the corporate tax rate), and in the user cost for motor vehicles in real form (prime rate minus PCPICE).

RDXF has two endogenous long-term international capital flow components: trade in outstanding Canadian bonds; and net new issues of bonds sold abroad. The trade in outstanding bonds depends on long-term Canada-U.S. interest differentials, the stock of Canadian bonds as a proportion of U.S. household wealth, the Canada-U.S. inflation differential, and the relative price of crude oil in U.S. dollars. Net new issues of bonds sold abroad, excluding federal government, is estimated as a proportion of PLH and corporate financial requirements as measured by the PLH deficit adjusted for the availability of C/QPP funds and business investment less retained corporate profits. The explanatory variables include interest differentials, the stock of bonds outstanding relative to U.S. GNP, the investment fraction of PLH current dollar expenditures, and the U.S. dollar relative price of crude oil (entering with a positive sign).

### **3.1.10 MACE**

The financial sector of MACE is very small. On the asset side it only contains equations for the demand for high powered money and for business assets. These equations are used in simulation to determine the market value of the business capital stock and the short-term interest rate. Alternatively, it is possible to treat the short-term interest rate as exogenous and allow high powered money to be determined by the equation. Since MACE has a government budget constraint, the change in high powered money is offset by a change in the opposite direction in government debt.

There is also an equation in MACE which sets the high powered money supply based on a reaction function reflecting income, the U.S. interest rate and the change in government debt.



The demand for high powered money (H) equation is:

$$\text{LOG}(H) = -2.0843 + .91060 * \text{LOG}(Y) - .12424 * \text{LOG}(rs)$$

where Y is nominal gross national product, and rs is the interest rate on 1-to-3 year Government of Canada bonds.

The income elasticity of the demand for high powered money is not too far from unity, but the interest elasticity is low.

The equation that is solved for the market value of non-energy capital (Vkb) is:

$$100 * Y_{\text{corp}} / (V_{\text{kb}} - V_{\text{res}}) = 10.196 + r_l + .82643 * (\text{rep}2 - \text{rtbw})$$

where Ycorp is non-energy corporate income after tax, Vres is the market value of residential housing, rl is the long-term government bond rate, rep2 is the earnings price ratio for U.S. equities, and rtbw is the yield on 91 day U.S. treasury bills.

According to this equation the value of the business capital stock adjusts so that the differential between the after-tax return on capital in Canada and the long bond rate tracks that in the U.S.

The long-term interest rate (rl) is explained as a function of Canadian and U.S. short-term rates.

$$r_l = .51024 + .33833 * r_l(-1) + .19751 * r_s + .49838 * r_{m2}$$

where rs is the Canadian short-term government bond rate, and rm2 is the rate on 5 year U.S. government bonds.

Money supply in MACE does not have a direct effect on price expectations. The financial sector impacts on the real sector in a number of ways. Depending on the exchange regime utilized, changes in interest rates and/or wealth can have an effect on the exchange rate and trade flows. This is the case for the limited intervention and nominal interest parity regimes. Other channels are also present. The ratio of average unit cost to output price for the non-energy sector, which incorporates an interest rate variable in the numerator, influences the timing of investment. The level is not directly affected by interest rates as the price of capital services, which is utilized in the choice of long-term factor proportions, reflects a constant real supply price of capital. This is the specification favoured by the data according to the model's builders. Wealth, which is a function of the market value of non-energy capital, is a determinant of consumption. Another link is the inclusion of interest costs in the price equation, the direct impact of which is to raise prices by 1 per cent for every 1 percentage point increase in the long-term interest rate.

International capital flows are explained by an equation for net foreign liabilities (excluding business assets) as a function of total OECD income measured in U.S. dollars and multiplied by the exchange rate to the .25 power to bring the currency mix of the numerator and denominator into line. This portfolio ratio is explained by its lagged value, the difference between Canadian and U.S. short-term rates, and a measure of temporary current account deficits or surpluses. According to the equation a 1 percentage point increase in Canadian short-term rates would raise net capital flows by \$3.8 billion in the first year, \$2.7 billion in the second, and lesser amounts thereafter.

This specification is utilized for three of the alternative exchange regimes: 1) the portfolio model with limited intervention; 2) the fixed exchange rate; and 3) purchasing power parity. Under nominal interest parity net capital inflows are determined from the balance of payments identity.

The MACE model also determines the non-resident owned proportion of the total capital stock. Retained earnings and revaluation adjustments increasing liabilities are calculated.

### **3.1.11 SAM**

SAM has four financial assets: 1) high powered money; 2) government bonds; 3) equity in the real capital stock; and 4) net foreign assets. Together they comprise financial wealth. Demands for these assets are specified as part of a constrained household portfolio within a Brainard-Tobin framework.

The supply of high powered money is determined by the monetary authorities. The supply of government bonds is determined, in the short-run, from the government budget constraint. In the long-run, however, taxation adjusts to reflect such things as interest on the debt, and there is not, necessarily, a perpetual deficit. The capital stock depends on cumulated investment decisions, and results in a flow of equities to finance the capital accumulation. Similarly, net foreign assets come from the cumulation of the balance of payments surplus.

The asset demand equations thus generate the before tax yield on government bonds, the price of equity, and the exchange rate. Since the four equation system is singular, the demand for high powered money is dropped in estimation and is not explicit in the model code.

The financial sector of SAM has very direct links to the real sector of the model. The expected money growth rate (minus steady state growth of output) influences expected consumer and product inflation which in the long-run drives wage and price inflation. The real side is also affected by the inclusion of wealth in the consumption function. Financial wealth is a component of non-human wealth. In addition to financial wealth, there is human wealth. Human wealth is dependent on the capitalization rate and after tax real wages and transfers from governments. The capitalization rate used in the present value calculations is linked to the financial system. An important monetary channel is the specification of a disequilibrium-money-balance effect on consumption. Money acts as a buffer stock for households. When a money shock hits the system,

it is gradually dissipated through transitory effects on consumption and certain other variables. Another link between the financial and real sectors is Tobin's q, which is based on the valuation ratio for equities, and which influences investment in the short-run.

A variable in the model that is very important in generating steady state results is the steady state after-tax return on the household portfolio. It rises in response to any gap between demand and steady state supply, and tends to decrease consumer demand because of its impact on wealth. This mechanism, combined with movements in the real exchange rate (influencing demand through trade), ensure the long-term consistency of aggregate demand and potential output.

International capital flows are modelled as part of an interrelated system of asset demand and supply in SAM. One of the important equations specified in terms of the percentage change in the stock of foreign-owned domestic capital (KWT).

$$\begin{aligned} \text{J1D}(\text{LOG}(\text{KWT})) &= \text{DNUCSS} + a_0 * (\text{CCRE} - \text{CCREUS} - a_1) \\ &+ a_2 * (\text{REQ} - \text{REQU} - a_3) + a_4 \end{aligned}$$

where DNUCSS is domestic potential growth,  
CCRE is the share of output available for capital,  
CCREUS is capital cost in the  
U.S., REQ is return on equity in Canada, and  
REQU is the return on equity in the U.S.

The equation is specified so that in equilibrium the stock of foreign-owned assets will grow at the same rate as potential (DNUCSS). The other two terms only count in disequilibrium. Term (1) is the capital cost gap between Canada and the U.S. and represents profitability in Canada relative to the U.S. The second term is the gap between the foreign and domestic return to equity, also exerting a positive impact on the growth of the foreign-owned capital stock.

To get net foreign investment in current dollars the change in the stock of foreign owned domestic capital must be multiplied by the investment price index.

The other main component of capital flows, the change in the net foreign assets of households, is derived from the balance of payments identity. There is also a corresponding stochastic asset demand equation for net foreign assets which constitutes another part of the complete system explaining international capital flows.

## **3.2 Wealth, Permanent Income**

### **3.2.1 CANDIDE 2.0**

There is no variable for wealth in CANDIDE 2.0. Only financial assets are explicitly modelled. Permanent income is also not defined.

### **3.2.2 TIM**

Personal wealth is defined in TIM. It is used to determine personal sector interest income. Permanent income is not specifically defined.

### **3.2.3 RDX2**

The original version of RDX2 had a variable for the market value of the end-of-quarter stock of total business fixed capital and inventories. To the resident owned part of this was added the value of the stocks of motor vehicles and other durables and of housing as well as the market value of government bonds. The result is the market value of private sector wealth. This variable did not have an important role in the model. Capital gains on the domestically owned portion of the total business capital stock, however, was included in the equation for consumer expenditures on services.

In the latest version of RDX2 wealth variables had no role in the consumption sector, but were utilized in the calculation of capital gains in the personal income tax sub-model.

In the original version of RDX2 disposable wage and non-wage income were treated differently in the equations for consumer expenditures. Real disposable wage income per capita was introduced with a seven quarter lag and real disposable non-wage income was entered contemporaneously. The lag weights varied from equation to equation.

In the latest version of RDX2 an explicit variable for permanent income was utilized. It was defined as a geometrically declining distributed lag on the sum of real disposable wage and permanent non-wage income per capita. Permanent disposable non-wage income was defined using interest and dividends from abroad, the real return on the stock of durables and housing, the real return on government bonds, and property income, and subtracting direct taxes on non-wage income.

### **3.2.4 CHASE**

Wealth is defined to include the residential and non-residential capital stock and financial assets. It is not utilized as an explanatory variable in the consumption functions but is used in asset demand equations. Permanent income is proxied in the consumption functions by taking a distributed lag on real disposable income minus the inflation premium on net personal wealth

and personal transfers to corporations. The CPI is used as the price deflator. The lag on real income varies from one consumption category to another as the lag structure is introduced through the inclusion of the lagged dependent variable.

### 3.2.5 DRI

The DRI model does not have a wealth variable. Permanent income is defined with respect to real disposable income per person 15 years of age and over adjusted to include provincial transfers to hospitals (YDN15&VR71) and deflated by the consumer deflator.

Permanent income has a long term growth component and a component reflecting changes in measured income. The specific formula is:

$$\begin{aligned} \text{YDPN15\&VR71} &= .88914 * 1.007854 * \text{YDPN15\&VR71}(-1) \\ &+ .11086 * \text{YDN15\&VR71} \end{aligned}$$

The long-term growth component of .785 per cent is the average compound rate of change of YDN15&VR71 over the 1953 to 1982 period. The adjustment coefficient of .88914 is said to be the quarterly equivalent of Friedman's annual rate of .625. The initial value of permanent income in 52Q4 was selected to give an average transitory income of zero over the 1953 to 1982 period.

### 3.2.6 FOCUS

There is no wealth variable in FOCUS. Permanent income is defined in relation to disposable income per household deflated by the implicit price index for consumer expenditures (and adjusted for the change in the treatment of hospital insurance). The formula utilized is:

$$\begin{aligned} \text{YDPERMHH71} &= 0.91667 * \text{YDPERMHH71}(-1) + (1.00636043^{**12}) \\ &* 0.833 * (\text{YD} + \text{VGL}@@\text{H} + \text{VGPV}@@\text{H}) / (\text{PC71VR} * \text{NHHOLDS}). \end{aligned}$$

where YDPERMHH71 is real permanent disposable income, YD is disposable income, VGL@@H is transfers from local governments to hospitals, VGPV@@h is transfers from provincial governments to hospitals, PC71VR is the implicit price index for total consumer expenditure, and NHHOLDS is the number of households.

Actual income affects permanent income with an average lag of 12 quarters.

### 3.2.7 MTFM

MTFM does not have a wealth variable. Permanent income is defined in terms of income adjusted for the impact of inflation and expected future taxes on wealth. It is deflated by the implicit

price index for consumer expenditures. The equation utilized is

$$YPERM = 0.1 * (YPD - YPDADJ)/PC + 0.9 * (1+B) * YPERM(-1)$$

where B is obtained from a five-year moving regression of the equation:

$$\text{LOG}((YPD - YPDADJ)/PC) = A + B * \text{TIMEQ}$$

This gives the average growth rate of income over the period. This specification is based on Darby (11).

The adjusted income concept is defined as:

$$YPDADJ = .5 \overset{(1)}{(YINTDIV - .5 * GIPD)} * PCER/RL10IND + .5 \overset{(2)}{* GIPD}$$

where YINTDIV is personal interest, dividends and miscellaneous investment income, GIPD is interest on the public debt, PCER is the expected rate of consumer price inflation as measured by an eight quarter moving average of actual inflation, and RL10IND is the McLeod, Young, Weir bond rate on ten industrial bonds.

Term (1) is an adjustment for the impact of inflation on financial wealth. The first .5 represents the extent to which the erosion of financial wealth by inflation is recognized. Term (2), which is also subtracted from interest and dividends in the inflation adjustment term, is another adjustment for the extent to which interest payments on the public debt are perceived likely to be financed through higher future taxes. This amount also is not counted as part of perceived disposable income and the corresponding financial wealth is not counted as perceived wealth. The .5 coefficient is an estimate of the proportion of debt the holders of which adjust their wealth to account for the possibility of higher taxes in the future.

The approach adopted in measuring and adjusting permanent income in MTFM goes beyond the other quarterly models in introducing the question of the degree to which interest income on the public debt should be regarded as being offset by future tax liabilities. This is an important question with direct bearing on the effectiveness of fiscal policy as an instrument of stabilization.

### 3.2.8 QFS

There are no wealth variables in QFS. Real per capita permanent income (deflated by the consumer expenditure deflator) is defined in accordance to the following formula:

$$\text{YPD71PK} = 0.22 * \text{YPD71/POP} + (1 - 0.22) * (1 + 0.0082) \\ * \text{YPD71PK}(-1).$$

where YPD71 is personal disposable income deflated by the consumer expenditure deflator, and POP is population.

The .82 per cent reflects the average quarterly growth rate of real per capita disposable income. The .22 coefficient on the current value of real per capita disposable income indicates a more rapid adjustment of permanent to actual income in QFS than in the DRI and FOCUS models where the coefficients are clustered around .1.

### 3.2.9 RDXF

Wealth does not have a large role in RDXF. It is only used in the determination of savings deposits at financial institutions and of current transfers from corporations. It is defined as the sum of the replacement cost of the real capital stock, including the stock of business capital, residential construction and business inventories, government debt (net of Government of Canada deposits), plus net foreign assets (measured on a book value basis as the cumulative value of the current account balance).

The income variable in RDXF is real effective purchasing power per capita. It is defined as personal disposable income minus personal transfers to corporations and the perceived inflation premium on interest receipts. The latter is calculated by multiplying a recognition coefficient of .5 times the product of expected inflation and the market value of the nominal stock of personal sector net financial wealth. The latter is equal to domestically held net government liabilities.

### 3.2.10 MACE

Private sector wealth has an important role in the MACE model. It or its components are utilized in the consumption equation, capital flows equations, and the financial sector. The broadest wealth measure is the market value of private sector wealth. It is defined as the sum of the market value on the stock of business fixed capital and inventories for both the non-energy and energy sectors, the net stock of government non-monetary liabilities, and high powered money, minus

net liabilities to non-residents.

Permanent income in MACE is not explicitly defined. Real wage income and wealth are included directly in the consumption function.

### **3.2.11 SAM**

Wealth also has an important role in SAM as the main driving variable of consumption. It is broken into two components, non-human wealth and wage and transfer wealth. The non-human wealth term is the sum of all financial assets owned by Canadian households including base money, government bonds (including treasury bills and CSBs), equities (including claims to housing), and net foreign assets. Wage and transfer wealth is defined as the present value of expected future real after-tax wage, unemployment insurance benefits and other transfer income. In SAM wealth also influences labour supply decisions and hence potential output in the model. It is also used as the scale variable in the long-run real-balance demand function, thus playing a role in the determination of the equilibrium price level. Permanent income per se has no direct role in SAM, and is instead replaced by its stock equivalent wealth.

## **3.3 Wages and Prices**

### **3.3.1 CANDIDE 2.0**

In CANDIDE 2.0 wages are modelled at the industry specific level. The base specification for the individual industry equations is an extended Phillips curve including an indicator of inflation expectations and an indicator of labour market tightness, the unemployment rate for prime age males. The latter is not included in all the equations, however, probably because it did not pass the estimation test. Some of the equations also incorporate U.S. wage rates and industry specific productivity as explanatory variables.

The use of the unemployment rate for prime age males as the labour market tightness variables can be taken to imply that the natural rate of unemployment is that consistent with the average rate of unemployment for prime age males.

Industry specific productivity is included in some of the equations, but more generally it is picked up by the constant term. There is no catch-up term in any of the wage equations.

The CANDIDE price expectations variable incorporating lagged consumer prices and the rate of change in the money supply is utilized in the wage equations.

The price determination process involves four steps: 1) the determination of sector prices; 2) conversion of sector prices into commodity output prices utilizing the 1971 input/output structure; 3) conversion of commodity output prices into final



demand component prices using the "Bridge Matrix" and indirect tax rates; and 4) adjusting these pseudo final demand prices for the constancy of input output coefficients.

Each industry value added equation is specified based on cost mark-up as a distributed lag on the industry wage rate, the percentage change in industry productivity, the percentage change in the industry user cost of capital, and the percentage change in the industry's value added deflator in the competing country, and the percentage change in industry output. The latter functions as a rough proxy for excess demand in some of the industries. In others changes in the average weekly hours serve as the excess demand variable. The sector deflators for non-commercial industries are modelled as wage rate equations.

Final demand prices are a weighted average of domestic and import prices. Sector value added deflators are converted into pseudo final demand prices based on the 1971 input/output content vector, and time series data on import prices are adjusted for variations in input/output relationships over time through an adjustment equation.

The long-term aggregate wage equation in CANDIDE has been characterized as:(12)

$$J1P(W) = 1.366 + .950 * J1P(.CPIE) + 10.75 * (1/DMURATE25.54) \\ + b * J1P(PROD) + c * J1P(WUS)$$

where J1P(W) is the percentage change in aggregate wages,  
 J1P(.CPIE) is the percentage change in expected inflation,  
 DMURATE25.54 is the unemployment rate for prime age males,  
 J1P(PROD) is the percentage change in labour productivity,  
 J1P(WUS) is the percentage change in the U.S. wage rate,  
 and coefficients are elasticities.

The aggregate value added deflator can be summarized as:

$$J1P(PX) = .65 * J1P(W) - .52 * J1P(PROD) + .10 * J1P(PM) \\ + .03 * J1P(PTE) + .15 * J1P(IUC)$$

where J1P(PX) is the percentage change in the aggregate value added deflator, J1P(PM) is the percentage change in the aggregate import price index, J1P(PTE) is the percentage change in the aggregate export price index, and J1P(IUC) is the aggregate user cost of capital.

The aggregate CPI can be summarized as:

$$\begin{aligned} J1P(CPI) = & .660 J1P(PX) + .157 * J1P(PM) + .108 * J1P(PT) \\ & + .075 J1P(FRMC) \end{aligned}$$

where J1P(CPI) is the percentage change in the CPI, J1P(PT) is the percentage change in the indirect tax rate, and J1P(FRMC) is the percentage change in the average mortgage rate.

Money has an impact on prices through its effect on the expected, rate of inflation in the wage equation as well as its effect on aggregate demand. The coefficient on money supply growth in the expected price equation is .27. Taking the wage and price equations together a 1 percent increase in the rate of growth of the money supply will increase the rate of inflation by .9 percent.

### **3.3.2 TIM**

The wage per worker is modelled in TIM for seventeen industries. The specifications can be characterized as a modified Phillips curve by industry tied to manufacturing, but with a role for industry specific labour market conditions such as hours worked and profits. In the aggregate the rate of inflation influences wages with a lag. After three years, although wages in some sectors such as retail trade, will have increased by less than the full amount of inflation, in the aggregate wages will fully reflect inflation. The key manufacturing rate equation has the inverse of the unemployment rate as an explanatory variable as do the equations for some other industries. Other industries are influenced indirectly by the unemployment rate as a result of the inclusion of the manufacturing wage in the wage equations.

The use of the aggregate unemployment rate in the wage equations can be interpreted as implying that the natural rate is the average unemployment rate over the sample period. This allows no role for demographic factors or unemployment insurance in altering the natural rate.

Long-term increases in wages above inflation are determined by productivity increases. These are modelled as increases in output per employee and increases in capital per worker.

Prices in TIM are determined as weighted combinations of industry value-added prices and foreign prices using an input/output system as is traditional for the CANDIDE family of models.

### 3.3.3 RDX2

The main wage equation in RDX2 is for quarterly earnings in mining, manufacturing, and other business (WQMMOB). The equation in the original version of RDX2 was one that established an equilibrium real wage dependent on productivity and the unemployment rate. The equation in the latest version of RDX2 was presented as either a factor share or as a Phillips curve with some refinements. By the time RDXF was constructed, the equation was just an extended Phillips curve. This shows the increasing dominance of the extended Phillips curve model.

The equation for WQMMOB in the latest version of RDX2 is:

$$\begin{aligned}
 J1P(WQMMOB) = & -4.5950 * QDBAD - 5.1289 * QDGOOD - .79208 * QC1 \\
 & \quad (1) \\
 & + .61245 * QC2 + .50506 * QC3 + 18.949 * ELEFF \\
 & \quad (2) \\
 & - 25.067 * J1L(.00093783 * (WQMMOB/PCPI)) \\
 & \quad (3) \\
 & + .30374 * J1P(HAWMM) + 8.8029 \\
 & \quad (4) \\
 & * ((NMMOBD - NMMOBS)/NMMOBS) + 6.6690 \\
 & \quad (5) \quad (6) \\
 & * J1L(.00093783(UGPPA/NMMOBD)) - .44541 * J1P(NMMOB)
 \end{aligned}$$

where QDBAD and QDGOOD are dummy variables for so called good and bad years for wage behaviour, QC1, QC2 and QC3 are constrained quarterly dummies, ELEFF is the labour efficiency factor in the production function, PCPI is the CPI, HAWMM is average weekly hours worked in mining, manufacturing, and other business, NMMOBD is the desired level of employment, NMMOBS is the potential labour force, UGPPA is private business product adjusted for unintended inventory accumulation, and NMMOB is actual employment in mining, manufacturing and other business.

The equation is basically one in which the wage rate adjusts to the gap between the real wage rate as dictated by productivity - term (1) plus term (5) - and the lagged real wage rate - term (2). Term (1) measures pure technological progress and term (5) measures productivity allowing for variations in the capital/output ratio. The third term captures increases in wages due to overtime and other factors not directly related to employment growth. The fourth term reflects labour market slack which affects the level of the equilibrium real wage. The sixth term allows newly hired workers to have a lower than average wage.

Another important private sector wage is quarterly earnings in construction. It is modelled similarly.

The price sector of RDX2 models eighteen disaggregated prices. The price equations are based on the medium- to long-run application of cost mark-up pricing behaviour. This is a specification common to all versions of RDX2. The typical form for a price equation is:

$$\begin{aligned}
 & \text{(1)} \\
 \text{LOG}(P/(1 + R)) &= a + b * \text{LOG}(WQMMOB * NMMOB/UGPPS) \\
 & \text{(2)} \\
 & + c * \text{LOG}((RCNR * KNCR + RCME * KME)/UGPPS) \\
 & \text{(3)} \\
 & + d * \text{LOG}((PFX * PF)) \\
 & \text{(4)} \\
 & + e * \text{LOG}(J4A(UGPPA/UGPPD)) \\
 & \text{(5)} \\
 & + f * \text{LOG}((J1L(KIB) - J12S(UGPP - UGPPA))/J1L(KIB))
 \end{aligned}$$

where P is a domestic price deflator, R is an indirect sales tax rate, RCNR and RCME are the user cost of capital for non-residential construction (KNRC) and machinery and equipment (KME), PFX is the exchange rate, PF is a foreign price deflator, and KIB is the stock of inventories.

The first term is for labour costs and the second for capital costs. The third term transmits international inflation or changes in the exchange rate into domestic prices. The fourth term reflects capacity utilization and thus introduces an impact of demand on the mark-up. The fifth term also represents demand pressure but with a stock dimension as measured by the difference between the stock of inventories and the cumulant of unintended inventory accumulation measured as a proportion of the inventory stock.

An important feature of the price sector which was considered to be unsatisfactory was its non-homogeneity with respect to costs. In particular the elasticity of prices with respect to capital costs was regarded to be much too low. It thus did not reflect the underlying Cobb-Douglas production function.

### 3.3.4 CHASE

The wage in the industrial composite, the key wage equation in the CHASE model, is characterized in the CHASE write-up as "a function of the marginal product of labour, the

difference between the actual and natural rate of unemployment, and a CPI term reflecting the attempt by workers to protect their purchasing power"(13). An inspection of the equation in the current version of the model revealed a quite different specification. It relates the percentage change in the wage bill plus supplementary labour income to the percentage change in current dollar gross private business product and the change in the gap between the actual and natural rate of unemployment. This specification is more akin to a wage share equation than a Phillips curve. The sum of the coefficients on the lagged percentage change in gross private business product in the equation is 1.0614. This is consistent with a moderately increasing wage share. The industrial composite wage is important in the CHASE model because all of the other wage series in the model are explained relative to it.

The natural rate of unemployment is a function of the percentage of the population 15 years and over which is between 19 and 24 and of real unemployment insurance benefits per capita. The natural rate series was constructed by CHASE.

Domestic prices are explained by mark-up equations. The costs included are average weekly wages in the industrial composite grossed up by supplementary labour income, the price of energy, excise taxes, manufacturers' sales tax, retail sales tax, import prices and total factor productivity. The cost variables used in the individual equations relate to the particular expenditure category to which the price applies. Capacity variables appear to be absent although the documentation suggests that they may have been employed in earlier versions of the model. The domestic price equations are specified so as to ensure homogeneity of degree 1 in costs.

### 3.3.5 DRI

The key wage rate in the DRI model, which is utilized in the price equations, is for average hourly earnings in manufacturing (AHM). It is a function of inflation (CPI/CPI(-4)) and the gap between the actual and full employment unemployment rate (RU-RUFE). The exact equation is:

$$AHM/AHM(-4) = .0474600 + JW(CPI/CPI(-4)) + JW(RU-RUFE)$$

The lag on inflation is 9 quarters long and the sum of the weights is approximately equal to 1. The lag on the labour market slack is two quarters and the sum of the weights is equal to .01, meaning that a 1 percent gap would lower the rate of wage inflation by one per cent per year.

The other wage variable is the national accounts average wage. It is also explained by an extended Phillips curve equation. This time, however, the equation also includes productivity as defined

by GNE per employed worker as an additional explanatory variable. The inflation term is also different, measured as a three quarter lag on the percentage change in the consumption deflator. The labour market variable is only included with a two quarter lag.

The full employment unemployment rate (RUFE) utilized in the gap in both wage equations is

calculated on the basis of research performed by D.P. Dungan and T.A. Wilson for the Economic Council.

The price sector of the model uses an input/output approach to determine 22 published industry prices including 14 manufacturing selling prices and 8 price indexes and wholesale prices for selected industries. Generated prices for industry and final demand categories are calculated using input/output weights. Federal manufacturers' and/or provincial sales taxes are also included. The labour cost variable in the generated price equations is unit labour costs in manufacturing calculated using the key average hourly earnings in manufacturing wage variable.

Industry selling prices are determined as a weighted average of foreign and domestic prices. The aggregate capacity utilization rate is also included to allow for the impact of demand.

The expenditure deflators and the CPI sub-components are a function of the corresponding generated price index, the relevant import price or U.S. price times the exchange rate, the provincial retail sales tax, where appropriate, and in some cases a measure of demand in the expenditure category.

### 3.3.6 FOCUS

FOCUS has one key wage equation. It determines the average annual wages and salaries per employee in the private sector (RCHAAWPS4Q) and is as follows for the pre-AIB period:

$$\begin{aligned}
 \text{RCHAAWPS4Q} = & \text{(1)} \\
 & .884834 + 2.49944 * (\text{J4A(RN)}/\text{J4A(RU)}) \\
 & \text{(2)} \\
 & + 2.49944 * (\text{J4D}(\text{J4A(RN)}/\text{J4A(RU)})) \\
 & \text{(3)} \\
 & + .525232 * (\text{J4A(PSALES71)} - \text{J4A(CPINSVR)}) \\
 & \text{(4)} \\
 & + .494169 * \text{J4A(RCHPEXP1)} \\
 & \text{(5)} \\
 & + .299837 * \text{J4A(CPINSVR)} \\
 & \text{(6)} \\
 & + (1.0 - .494169 - .299837 * \text{J4A(CPINSVR)}) \\
 & \text{(7)} \\
 & + .0529587 * (\text{J4A(CPINSVR)} - \text{J4A(RCHPEXP1(-1))})
 \end{aligned}$$

where CPINSVR is the CPI, PSALES71 is the price of final sales, RCHPEXP1 is the change in the CPI expected one year in the future, RN is the natural rate of unemployment calculated by Dungan and Wilson using a methodology similar to that employed by the Department of Finance for the cyclically

adjusted rate of unemployment, and RU is the unemployment rate .

The first term is the ratio of the natural rate to the actual rate of unemployment, which is an inverse measure of labour market slack. The second term introduces an additional effect on wages from changes in slack. The third term allows for a differential effect between output prices which increase employers ability to pay and the CPI. The fourth term captures expected inflation one year in the future as constructed using a regression of actual future inflation on a set of contemporaneous explanatory variables. The fifth term permits actual inflation to have an impact on wages. The final term which is the difference between actual and expected inflation is for catchup.

The key price in FOCUS is the implicit deflator for privately produced GNP(PGNPP71). It can be determined either as a market clearing price equating the demand and supply for real private domestic production or by a stochastic equation based on cost mark-up. This equation is:

$$\begin{aligned}
 & (1) \\
 \text{J1D(PGNPP71)} = & \text{PGNPP71}(-1) + 1.21925 * \text{J1D(ULCN +UTXN)} \\
 & (2) \qquad (3) \\
 & + .102534 * \text{J1D(UIMP)} + .381317 * \text{J1D(ULCA - ULCN)} \\
 & (4) \\
 & + 1.20024 * \text{J1D(RTDSALES - UTXN)} \\
 & (5) \\
 & - .0703661 * ((\text{KINVEAF71}/\text{GNPP71}) \\
 & - (.781 + .00026 * \text{DUMYTIME})) \\
 & (6) \\
 & + .0262511 * \text{J1D(POIL}/3.15) \\
 & (7) \\
 & + .00898549 * \text{J1D(POIL}(-1)/3.15)
 \end{aligned}$$

where ULCN is normal unit labour costs defined as normal productivity times normal wage costs including employer contributions, ULCA is actual unit labour costs, UTXN is normal unit sales tax rates, UIMP is normal unit labour costs, RTDSALES is ad valorem taxes per unit of GNP, KINVEAF71 is real non-farm business inventory at the beginning of the quarter, and POIL is the crude petroleum price.

The first term is for normal unit labour and tax costs. Its coefficient indicates a 22 percent mark-up. The second term is for import costs. Its coefficient may appear small, but it reflects only the impact of import prices on domestic production.

The various final expenditure deflators in FOCUS include import prices. The third term allows for some effect of actual unit labour costs. The fourth term discriminates between the impact of ad valorem taxes and other sales taxes. The fifth term for the deviation of the inventory-to-sales ratio from its trend provides for a downward pressure on price from greater than normal levels of inventory stocks. The sixth and seventh terms are for the price of oil. That it is included with such a short lag implies a rapid pass through on energy prices that is viewed as so surprising by the model builders that an alternative equation is offered excluding energy prices.

### 3.3.7 MTFM

The key wage variables in MTFM are for average weekly wages and salaries. Equations are included for average weekly wages and salaries in agriculture, other primary, manufacturing, construction, services, and public administration and defense. Equations are also included for wages and salaries per employee. A representative equation is for manufacturing:

$$\begin{aligned} & \text{(1)} \\ \text{LOG(WRAWWMAN/AWHMAN)} &= .10440 * \text{J4A(PISMAN} * \text{QMAN)} \\ & \text{(2)} \\ & + (1 - .10440 - .72743) * \text{LOG(PC(-1)} * (1 + \text{PCER/100)} \\ & \text{(3)} \qquad \qquad \text{(4)} \\ & - .00329 * \text{J4A(RIBP(-2))} + .72743 * (\text{WRAWWMAN(-1)/AWHMAN(-1)}) \end{aligned}$$

where WRAWWMAN is average weekly wages and salaries in manufacturing,  
 AWHMAN is average weekly hours in manufacturing,  
 PISMAN is the industry selling price in manufacturing,  
 QMAN is the real output price in manufacturing, PC is the  
 consumer price deflator, PCER is expected inflation on the  
 consumer price deflator, and RIBP is the real interest rate  
 measured as the rate of long-term industrial bonds minus PCER.

This equation was estimated in first difference form, but is given in levels form to simplify the presentation. The equation is not a Phillips curve specification. Instead it is based on the macroeconomic theory of wage behaviour as suggested by Lucas and Rapping (14). Under this approach, both a demand and supply function for labour are specified and the wage rate is determined so as to equilibrate demand and supply.

Term (1) in the equation reflects the nominal value of output in manufacturing. Term (2) is the expected price level. Term (3) is the real interest rate. Term (4) is the lagged wage variable and is included to allow for lagged adjustment. In the long-run, under this specification wage growth is a weighted average of nominal output growth and consumer price growth, and the change in the real interest rate. If manufacturing and consumer prices rise at the same rate, the elasticity of wages with respect to prices is unity.



The price sector of MTFM is a stage of processing type of model with raw material prices, industry prices, and final demand prices determined largely recursively, but with prices set on world markets and import prices entering in at each stage.

The raw materials prices correspond to those in Statistic Canada's raw material price index. Industry prices are for manufacturing, construction and electrical power. The specifications are cost mark-up. Also included to modify the mark-up model are measures of excess demand such as the rate of capacity utilization in manufacturing and the percentage deviation of output from trend. The coefficients for factor shares are estimated from input/output data. The material cost variables are also constructed from various prices, taxes and input shares.

There are 12 consumer deflators in MTFM. Consumer expenditures are composed of commodities produced by domestic manufacturers, imported commodities, trade margins, and net indirect taxes. The consumer deflators have the following general form:

$$\begin{aligned}
 J1D(PC/1 + WTR * RTIRST) = & C0 + C1 * J1D(LOG(1 + WTM * RTIFSG) \\
 & * PIMC)) + C2 * (F1 * J1D(LOG(PIMC)) \\
 & + F2 * J1D(LOG(WS/(((PRODTCS**B) \\
 & * (PRODCS**1-B)))))) + (1 - F1 - F2) \\
 & * J1D (LOG(JW(UCS)) + C3 * EDS
 \end{aligned}$$

where PC is the consumer deflator for a category, WTR and WTM are the proportion of a particular category subject to the retail sales tax rate RTIRST, and the manufacturers' sales tax rate RTIFSG, PIMC is materials cost, WS is labour income per person-hour in the service industry, PRDTCS and PRODCS are output per person-hour variables for the service industry, UCS is user cost in the non-energy service industry, and F1 and F2 are factor shares in the retail and wholesale trade industry.

The other domestic demand deflators are for residential construction, investment in machinery and equipment and non-residential construction and government spending. The equations for these deflators are similar to those for industry prices and consumer deflators.

### 3.3.8 QFS

The key wage equation in QFS is for the average wage per paid employee (WAYAI). It is:

$$\begin{aligned} \text{J1P(WAYAI)} = & \text{(1)} \quad \text{(2)} \\ & -1.22 - .60 * \text{QAIB} + 1.62 * \text{LFURGAPP} + .25 \\ & \text{(3)} \quad \text{(4)} \\ & * \text{J1P(WAHR)} + .83 * \text{J1P(LFPROV16)} - .14 \\ & \text{(5)} \quad \text{(6)} \\ & * \text{J1P(LPEP)} + \text{PCPIE8X} \end{aligned}$$

where QAIB is a dummy variable for the AIB, LFURGAPP is the natural rate of unemployment divided by the actual, WAHR is the average weekly hours worked in manufacturing, LFPROV16 is a 16 quarter moving average of the ratio of real gross national expenditure to total employment, LPEP is paid employment, and PCPIE8X is the expected increase in the consumer price index as measured by an eight quarter moving average of the actual increase in the CPI.

The first term is a dummy variable for the AIB. It is equal to .25 in 1975Q4 and 1976Q1, .5 in 1976Q2 and Q3, .75 in 1976Q4, 1 from 1977Q1 to 1978Q1, .75 in 1978Q2 and .5 in 1978Q3. The sum of these values is 8.5. The coefficient on the AIB dummy is .6. This indicates that the direct effect of the AIB was to lower the wage level by 5.1 per cent by the fourth quarter of 1978.

The second term measures labour market slack as measured by the ratio of the actual unemployment rate to the NAIRU. If the NAIRU were assumed to be 6 per cent, the coefficient would suggest that a 1 per cent decline in the unemployment rate from 11 to 10 per cent would raise the rate of increase of wages by .88 per cent. The impact of a change in the rate of unemployment would be greater at lower levels of unemployment.

The third term representing average weekly hours in manufacturing is included to pick up the effects of overtime on average wages. It could also be interpreted as an additional demand pressure variable.

The fourth term captures the impact of productivity on wages. The fifth term for the percentage change in paid employment makes an allowance for the extent to which new workers earn less than average wages due to seniority and skill progression. The sixth term represents expected inflation as measured by the consumer price index. Its coefficient has been constrained to equal 1 as suggested by the theory of the inflation-augmented Phillips curve.

The price equations in QFS are based on a mark-up pricing model. The costs considered include unit labour costs, material costs and import prices. The indirect tax rate is also included. The equations are estimated in first difference form and sometimes include variables to reflect excess demand, such as excess inventories or final demand relative to desired output. QFS has a high degree of disaggregation of CPI categories. National accounts consumption deflators are generated using the CPI components.

### 3.3.9 RDXF

The RDXF wage equations follow an expectations augmented Phillips curve equation. The two key wage series are average weekly wages in the industrial composite (WNIC) and average weekly wages in community services, government and agriculture (WOTH). The industrial composite equation is:

$$\begin{aligned}
 & \text{(1)} \\
 \text{J1D}(\text{LOG}(\text{WNIC})) &= 1.43530 * \text{J12A}(\text{J1D}(\text{LOG}(\text{ETFP}) - .00165 \\
 & \text{(2)} \\
 & * (\text{RU} - 1.92915 * \text{J6A}(\text{NPER.GE.19754.})) - .00969 \\
 & \text{(3)} \\
 & * (\text{NPER.GE.19761.}) * \text{NPER.LE.19774.}) \\
 & \text{(4)} \\
 & + \text{J8W}(\text{J1D}(\text{LOG}(\text{J1L}(\text{PCPI})))) + .37523 \\
 & \text{(5)} \\
 & * \text{J7A}(\text{J1D}(\text{LOG}(\text{J1L}(\text{PGPP}/\text{PCPI}))))
 \end{aligned}$$

where ETFP is a measure of trend factor productivity derived from a Cobb-Douglas production function, RU is the difference between the actual unemployment rate and the rate at trend output, NPER is the period number, PCPI is the consumer price index, and PGPP is the price deflator for private business product.

The first term allows the changing productivity growth to be explicitly modelled. The second term is the labour market gap variable measured as the gap between actual and the trend unemployment rate. The trend unemployment rate is calculated by a two step procedure. First, the actual unemployment rate is regressed on an output gap, and two structural variables representing unemployment insurance and the proportion of the young people in the labour force. Second, the gap variable is set equal to 1 giving the unemployment rate at trend output. The third term is an AIB dummy. The fourth term reflects price expectations as measured by an eight quarter lag on actual inflation. The sum of the weights was constrained to unity. The fifth term allows output prices to have a differential impact on wages from the consumer price index

reflecting labour demand considerations.

The other wage variable (WOTH) is tied to the industrial composite wage by an equation incorporating a number of dummy variables to explain the difference.

The consumer price index is the key price in RDXF. It is modelled as a near identity adding up the nine consumer expenditure deflators using fixed expenditure weights. The individual deflators are specified as producer prices and follow a flexible mark-up on costs. The general form of the equations is:

$$\begin{aligned} (P - (1 + RT)) = & a_0 + a_1 * ULC + a_2 * UKC + a_3 * PT \\ & + a_4 * PENERG + a_5 * POTH + a_6 * CAPU \\ & + a_7 * (P(-1) - (1 + RT(-1))) \end{aligned}$$

where RT is an indirect sales tax rate, ULC is normalized unit labour costs, UKC is normalized unit capital costs, PT are trade prices, PENERG is the price of energy, POTH are other costs including property taxes, farm gate prices and financial costs, and CAPU is a measure of capacity utilization in level form (UGPP/UGPPD).

### 3.310 MACE

The MACE wage equation is:

$$\begin{aligned} J1P(W) = & \overset{(1)}{.62829} * J1P(W(-1)) + \overset{(2)}{.37171} * J1P(Pa) \\ & \overset{(3)}{+ .26602} * (J1P(Pxne/Pmne)) \\ & \overset{(4)}{+ .31392} * (J2A(J1P(Q/Qsv))) \\ & \overset{(5)}{+ .37171} * J1P(LPI) \\ & - .035186 * D77 - .021573 * D78 \end{aligned}$$

where W is the average annual wage, Pa is the price of absorption, Pxne is the export price index, Pmne is the import price index, Q is the gross output, QSV is supply, and LPI is a labour productivity index.

The second term in the equation reflects the prices perceived by workers. Its long-run coefficient has been constrained to unity, thus making nominal wages rise in step with prices. The third term for the terms of trade represents businesses' ability to pay or profitability. The fourth term is the change in output relative to supply and is a demand measure. The fifth term is productivity of industry. Since it has been constrained to be one minus the coefficient on the lagged wage, the long-run real wages will keep pace with productivity.

The two key price equations in MACE are for the implicit price index for output (Pq) and the implicit price index for absorption (Pa). The price index for output is specified:

$$\begin{aligned}
 & \text{(1)} \\
 \text{J1P(Pq)} &= -.0065327 + .87508 * \text{J1P(Ckew)} \\
 & \text{(2)} \\
 & + .12492 * \text{J1P(Pw)} \\
 & \text{(3)} \\
 & - .054684 * (\text{J2A(Kinv)} - \text{Kinv}^*)/\text{Kinv}^*
 \end{aligned}$$

where CKew is equilibrium unit costs for capital, labour, and energy,  
Pw is an index of world prices, Kinv is the stock of  
inventories, and Kinv\* is the desired stock of inventories.

The first term reflects domestic costs. The second term the transmission of international inflation. The third captures demand pressures as measured by the gap between actual and desired inventories.

The second price equation is for the price of absorption:

$$\begin{aligned}
 \text{J1P(Pa)} &= .0031536 + .80485 * \text{J1P(Pq)} * (1 + \text{rti}) \\
 & + .15825 * \text{J1P(Pmne)}
 \end{aligned}$$

where rti is the indirect tax rate on the non-energy sector,  
and Pmne is the price of imports of non-energy goods  
and services.

The price of absorption is simply a weighted average of output prices adjusted for indirect taxes and the price of imports.

### 3.3.11 SAM

A simplified version of the main SAM wage equation is:

$$J1D(\text{LOG}(w)) = a * \text{LOG}(ws/w) + b * (\text{RNAT} - \text{RNU}) + \text{DNPX} + \text{DNPRX}$$

where  $w$  is the wage rate,  $ws$  is the steady state wage,  $\text{RNAT}$  is the natural rate of unemployment,  $\text{RNU}$  is the actual unemployment rate,  $\text{DNPX}$  is the underlying rate of inflation, and  $\text{DNPRX}$  is trend real wage growth arising from productivity gains.

The equation expresses the rate of change of wages as a function of the extent of disequilibrium in the level of money wages, the gap between the natural and actual rate of unemployment, and trend terms.  $\text{DNPX}$  can be viewed as capturing inflationary expectations and fundamental determinants of the equilibrium rate of inflation. In the long run wages are determined by the steady state wage which depends on technology and factor prices. This steady state wage is consistent with full employment of labour and the zero-excess-profit condition. In the shorter-run the wage equation exhibits Phillips-curve-like behaviour. The estimated coefficient on the gap implies that a 1 percentage point unemployment rate gap would lower the rate of increase of wages by 2 per cent. This is by far the largest effect in any of the models.

The main price equation in SAM is for the price of the domestic non-energy good (PD). A simplified version of this equation is:

$$J1D(\text{LOG}(\text{PD})) = a * \text{LOG}(\text{PS}/\text{PC}) + b * \text{LOG}(\text{SALES}/\text{UGPCSS}) + c * \text{LOG}(\text{INVCD}/\text{INVC}) + \text{DNPX}$$

where  $\text{PC}$  is the consumption price index,  $\text{PS}$  is the steady state consumption price,  $\text{SALES}$  represents the current state of real aggregate demand from various sources in a flow dimension,  $\text{UGPCSS}$  is potential output in the non-energy sector,  $\text{INVCD}$  is the level of the desired inventory stock,  $\text{INVC}$  is the actual inventory stock, and  $\text{DNPX}$  is the underlying rate of inflation.

The rate of change of  $\text{PD}$  depends on three factors: 1) price disequilibrium as measured by term (1); 2) excess demand as represented by the sales to potential output ratio (term (2)) and the ratio of the desired to actual inventory stocks (term (3)); and 3) the underlying rate of

inflation as determined by expectations and fundamental considerations. Steady state inflation for the consumption price index depends on money supply growth. The inflation rate for the domestic non-energy good also tends toward this value in the long run.

### 3.4 Aggregate Supply

#### 3.4.1 CANDIDE 2.0

CANDIDE 2.0 is built upon an input/output framework with a 48 industry level of disaggregation. Input/output models are based on a Leontief technology with fixed coefficients of production and one primary input, labour. CANDIDE does not specify potential output. Output is not subject to any long-run production function constraints. Discrepancies can exist between expenditure and production.

The investment equations of CANDIDE are based on Jorgenson's neoclassical model and thus implicitly assume Cobb-Douglas technology. The man-hours equations are based on either Cobb-Douglas or CES technology. Consistency is not imposed on the various implicit production functions by any framework of interrelated factor demands. This would be a very difficult task in practice, but not impossible in principle, in a model based on an input/output structure.

Private investment in CANDIDE is disaggregated into 38 industries and for each industry into non-residential construction and machinery and equipment. The specification of the equations follows that of Jorgenson and is derived from neoclassical investment theory assuming profit maximization and lagged adjustment of actual to desired capital stocks. The key explanatory variables are all industry specific. They include: 1) real output; 2) price; 3) the user cost of capital; and 4) capital stock. The form of the equation is:

$$I_i = a_0 + JW(J1D(P_i * X_i/IUC_i)) + JW(IK_i)$$

where  $I_i$  is investment in industry  $i$ ,  $P_i$  is the price of industry  $i$ ,  $X_i$  is the output of industry  $i$ ,  $IUC_i$  is the user cost of capital in industry  $i$ , and  $IK_i$  is the capital stock in industry  $i$ .

Sometimes the relative cost and output terms are entered in the investment equations separately.

The user cost of capital in CANDIDE is industry specific and provides a useful way to incorporate industry specific information on depreciation, the corporate tax rate, the investment tax credit, and capital consumption allowances. The user cost equations are of the form:

$$IUC_i = ((1/(1 - IET_i)) * PF_i * (FRATE + IED_i)) \\ * (1 - IZ_i * IET_i) * (1 - ITC_i)$$

where  $IET_i$  is the effective tax rate on industry  $i$ ,  $PF_i$  is the final demand deflator of the investment category,  $FRATE$  is the nominal yield on long-term industrial bonds,  $IED_i$  is the economic depreciation rate in industry  $i$ ,  $IZ_i$  is discounted capital cost allowances in industry  $i$ , and  $ITC$  is the effective tax rate in industry  $i$ .

Note that the nominal interest rate is utilized in the user cost calculation.

Private inventory demand in CANDIDE 2.0 is disaggregated into seven holding industries: 1) agriculture; 2) forestry; 3) mining; 4) retail and wholesale trade; 5) durable manufacturing; 6) non-durable manufacturing; and 7) other. Equations are included for all of the industries except agriculture which is exogenous. Inventory change equations follow the accelerator model and are a function of: 1) activity levels in the holding industry; 2) the lagged inventory stock of the holding industry; and 3) the rate of change of prices relevant to the holding industry.

Employment demand in CANDIDE 2.0 is broken down into 37 industries and expressed in terms of man-hours. The man-hours equations are either: 1) renormalized Cobb-Douglas production functions with lagged adjustment; or 2) labour demand functions derived from the first order equilibrium conditions of CES production functions. In the first case the explanatory variables are industry specific output and capital stock; in the second industry specific output and the real wage. A time trend is also often included.

Average weekly hours equations are provided in CANDIDE 2.0 to translate man-hours into employment. They are a function of the industry specific after-tax real wage, the unemployment rate and in some cases a time trend.

### **3.4.2 TIM**

TIM like CANDIDE embodies a detailed input/output production sector. There is no overall production function. Potential output is not defined. Output is not subject to any long-run production function constraints, but is primarily demand driven. Short-term productivity ensures that output plus inventories and net exports equals sales.

The investment equations of TIM are based on Jorgenson's neoclassical model with its assumed Cobb-Douglas production function. The employment equations are also mostly based on inverted Cobb-Douglas production functions. The explicit or implicit parameters of the production functions, however, are not necessarily the same in the investment and employment equations. The level of disaggregation is different between investment and employment in most cases, precluding constraints on parameters across equations.



Investment in TIM is modelled for over 45 sectors and for both machinery and equipment and non-residential construction. The investment equations are generally based on the Jorgenson neoclassical model. The functional form of the estimated equations is:

$$I_i = a_0 + a_1 * J1D(Y_i * P_i/C_i) + a_2 * K_i(-1)$$

where  $I_i$  is investment in industry  $i$ ,  $Y_i$  is output of industry  $i$ ,  $P_i$  is the price of output of industry  $i$ ,  $C_i$  is the user cost of industry  $i$ , and  $K_i$  is the stock of capital of industry  $i$ .

The user cost variable ( $C_i$ ) is defined as

$$C_i = P_{Gi} * (r + d_i)$$

where  $C_i$  is the user cost of capital in industry  $i$ ,  $P_{Gi}$  is the price of investment in industry  $i$ ,  $r$  is the interest rate on industrial bonds, and  $d_i$  is the depreciation rate in industry  $i$ .

There are two things worthy of note here. First, a nominal interest rate is used as the measure of the cost of financing. Second, the rental cost is specified before tax and does not have built-in tax parameters.

Other investment specifications utilized in TIM, but much less widely, relate investment to output and the interest rate or to output and the lagged capital stock.

Inventory change is modelled in TIM for eight categories: 1) wholesale trade; 2) grain; 3) other farm; 4) non-farm primary; 5) durable manufacturing; 6) non-durable manufacturing; 7) cars at the retail level; and 8) other retail trade. The equations are based on a stock adjustment model in which inventory change is related to the gap between desired and actual inventory stocks. The desired stock is primarily a function of some activity variable such as real output, or consumption of relevant goods and services. The nominal interest rate on commercial paper is included as an additional explanatory variable in the two equations for the change in manufacturing inventories. Dummy variables reflecting special factors are also in some of the equations.

Employment demand in TIM is disaggregated into 23 industries. Most of the equations are based on a stock adjustment model with the desired employment derived by inverting the Cobb-Douglas production function for the industry. The average age of the capital stock is introduced as a proxy for technological change and increased efficiency. A representative equation is that for non-durable manufacturing:

$$\begin{aligned}
 & \text{(1)} \\
 \text{J1D}(\text{LOG}(\text{MANDET})) &= a_0 + a_1 * \text{D75} + a_2 * \text{MNDVMK} \\
 & \text{(2)} \\
 & + a_3 * \text{LOG}(((\text{MNDCMK} + \text{MNDCCCK}) * ((\text{MANDY}/ \\
 & (\text{MNDCMK} + \text{MNDCCCK}))^{1/.66})/\text{MANDET}(-1))
 \end{aligned}$$

where MANDET is employment in non-durable manufacturing, D75 is a dummy variable equal to 1 in 1975, MNDVMK is the vintage of the capital stock of machinery and equipment in non-durable manufacturing, MNDCMK is the stock of machinery and equipment in non-durable manufacturing, MNDCCCK is the stock of non-residential construction in non-durable manufacturing, and MANDY is real domestic product in non-durable manufacturing.

Term (1) measures the vintage of the capital stock. It is calculated as a distributed lag on a time trend with the weights given by the share of investment minus scrappage in the period to the total stock of the particular type of capital. The coefficient on this term, which can be viewed as a time trend, is negative. Term (2) represents the gap between desired and actual employment. A feature of this specification is that if output and capital stock grow at the same rate, employment will grow less rapidly due to the negative coefficient on the vintage of the capital stock. This allows for labour augmenting technical change and productivity growth.

In some industries other formulations are utilized. For instance, employment in pipelines is tied to capital stock and employment in agriculture is positively related to overall unemployment as well as to agricultural output.

Total annual hours worked by industry in TIM depends on employment in the industry and on trend and cyclical factors. The cyclical variables most commonly utilized are output and productivity.

### 3.4.3 RDX2

The key output concept in RDX2 is gross private business product excluding agriculture and commercial services (UGPP). The supply of output is based on a three factor Cobb-Douglas production function with the stock of machinery and equipment, the stock of non-residential construction, and the labour inputs in efficiency units. Labour input in efficiency units is defined as the product of employment in mining, manufacturing and other business and average weekly hours times a labour efficiency factor. The economy quite reasonably allows for variable operating rates and is not required to operate on its production function in the short-run. In the longer-run, however, it exhibits a tendency to gravitate back on its production function.

The RDX2 concept corresponding to potential output (UGPPD) is calculated using the same production function but with employment based on the average employment rate and with trended weekly hours. Another important output concept in RDX2 is private business product adjusted to remove unintended inventory changes (using the coefficient from the inventory change equation). This concept is called UGPPA. The ratio of UGPPA to UGPPD is the RDX2 measure of capacity utilization which appears in some of the price and import equations.

Investment in RDX2 is forward looking. Investment in machinery and equipment is specified to be a function of the gap between future output and preferred output according to the vintage stock of capital and the desired capital output ratio calculated from the production function assuming cost minimization. A cash index influences the speed with which the gap is closed. Lagged investment is introduced as an additional explanatory variable.

Investment in non-residential construction also depends on a forward looking gap variable and the lagged capital stock. However, dependency is intermediated by an equation for commercial, industrial, and engineering contract awards. Investment in non-residential construction is a distributed lag on real contract awards.

The imputed rental prices for machinery and equipment and non-residential construction utilize a weighted sum of the real supply price of capital in Canada and the U.S. based on ownership of the domestic capital stock as the interest rate variable. They incorporate the weighted corporate tax rate, and the present value of capital consumption allowances for each category. They also allow for the deductibility of interest payments on the debt financed portion of investment.

Employment in mining, manufacturing, and other business in RDX2 can be viewed as the residual factor of production. It is modelled as a function of the gap between desired employment calculated by inverting the production function given output, capital stocks, and long-run hours. The desired level of employment is constrained by a non-linear relationship to the potential labour force. The gap between desired quantity of labour and labour supply is utilized in the wage equation as the labour market tightness variable.

The gap between average weekly hours worked in mining and manufacturing and trend hours as a per cent of trend hours is explained as a function of the gap between constrained desired employment and actual employment divided by actual.

The RDX2 inventory change equation is fairly simple. Intended inventory change depends on the gap between the desired and actual inventory stocks. The desired stock is defined as a distributed lag on sales and a separate lag on imports. The unintended component is represented by the difference between the supply of output calculated from the production function and real private business product excluding inventory change. It is the coefficient on this term that is utilized to adjust private business product for unintended inventory accumulation.

The use of the production function rather than "expected sales" to define the buffer change in inventories is a crucial difference between RDX2 and any other Canadian models prior to MACE and SAM (15).

Factor demands in RDX2 are consistently estimated within a framework of interrelated factor demands. There is a hierarchy of demands running from short-run productivity, hours, employment, machinery and equipment, and non-residential construction. In RDX2 there is no powerful mechanism to ensure that the economy returns to the production function within a short period of time, although the model exhibits a tendency in this direction. Short-run productivity changes can persist for fairly long periods of time. Whether or not this is an accurate description of the real world is an empirical question. There is much empirical evidence that variable operating rates are an important characteristic of the economy which can not be ignored. There is less reliable evidence about the duration of variable operating rates.

### 3.4.4 CHASE

The key output variable in the CHASE model is for real private, business output which is defined as real GNE excluding capital consumption allowances, external interest and dividends, output of the farm and government sectors, and gross rent. The production function utilized in the CHASE model to calculate aggregate supply is:

$$XGPPSCA = (KMEXCA(-1)**EKMECA(-1)) * (KNRCXECA(-1)**EKNRCA(-1)) \\ * ((NICCA * NAWMMCA * .052)**ELCA) * ETFPCA$$

where XGPPSCA is aggregate supply, KMEXCA is the stock of non-farm machinery and equipment excluding energy, EKMECA is the production function coefficient on M&E, KNRCXECA is the stock of non-farm non-residential construction excluding energy, EKNRCA is the coefficient on non-residential construction, NICCA is industrial composite employment, NAWMMCA is average weekly hours worked in manufacturing, ELCA is the coefficient on labour, and ETFPCA is total factor productivity in the private business sector.

There is no requirement that demand and supply as defined by the production function be equilibrated in the long-run.

The same production function is used with the substitution of trend hours and trend labour demand to define trend private business product. It is the ratio of private business product to trend private business product that serves as the capacity utilization variable in the CHASE model which appears most importantly in the import equations.

The investment sector has equations for non-energy investment in business non-residential construction, and machinery and equipment. They are basically accelerator equations with a cash flow variable added. There are no cost of capital variables embodying tax parameters. Energy investment is treated as exogenous.

The investment in non-residential construction (INRCCA) equation is:

$$\begin{aligned}
 & \text{(1)} \\
 \text{LOG(INRCCA - INRCNRCA)} &= .70520 - .98071 * \text{LOG(KNRCXECA(-1))} \\
 & \text{(2)} \\
 & + .13440 * (\text{LOG(YCCA(-1)) - TCCFCA(-1) + CCACZCA(-1)}) \\
 & \text{(3)} \qquad \qquad \text{(4)} \\
 & - \text{LOG(XGPPZCA)} + .26989 * \text{LOG(INRCNRCA)} \\
 & \text{(5)} \\
 & + \text{JW}(\text{LOG(XGPPCA(-1))})
 \end{aligned}$$

where INRCNRCA is energy investment in non-residential construction, KNRCXECA is the stock of non-farm nonresidential construction excluding energy, YCCA is corporate profits before tax, TCCFCA is federal corporate income tax collections, CCACZCA is capital consumption allowances of corporations, XGPPZCA is gross private business product in current dollars, and XGPPCA is real gross private business product.

Term (2) is the cash flow variable. RDX2, the DRI model and FOCUS also have cash flow terms in their investment equations, but cash flow has a larger role in CHASE than in the other models. The sum of the lag weights on gross private business product (term (5)) is 1.098.

The investment in machinery and equipment (IMECA) equation is similar:

$$\begin{aligned}
 \text{LOG(IMECA - IMENRGCA)/KMEXECA(-1)} &= -.51047 - .28077 \\
 & * \text{LOG(KMEXECA(-1)/XGPPCA(-1))} + .65756 \\
 & * \text{LOG}((\text{IMECA(-1) - IMENRGCA(-1)})/\text{KMEXECA(-2)}) \\
 & + .16822 * (\text{LOG(YCCA(-1)) - TCCFCA(-1)} \\
 & + \text{CCACZCA(-1) - LOG(XGPPZCA(-1))})
 \end{aligned}$$

where IMECA is investment in machinery and equipment,  
 IMENRGCA is energy investment in machinery and equipment,  
 KMEXECA is the stock of non-farm machinery and equipment  
 excluding energy, and the other variables are as defined above.

The main employment variable in the CHASE model is employment in the industrial composite. It is the variable that appears in the production function. Other employment categories such as non-commercial services and public administration are also modelled. As are categories for paid and unpaid, part-time and full-time employment and by age and sex.

Employment in the industrial composite depends on nominal private business product divided by the hourly labour cost, a proxy for the marginal product of labour. The equation is:

$$\text{LOG}(\text{NICCA}/\text{NICCA}(-1)) = -.19083 + .17124 * (\text{LOG}(\text{XGPPZCA}/\text{WNICCA} \\ * (1 + \text{RYWSLPCA}/\text{NAWMMCA})) - \text{LOG}(\text{NICCA}(-1)))$$

where NICCA is employment in the industrial composite,  
 XGPPZCA is gross private business product, WNICCA is average  
 weekly wages and salaries for the industrial composite,  
 RYWSLPCA is the proportion of supplementary income in total  
 labour income, and NAWMMCA is average weekly hours worked  
 in manufacturing.

The change in non-farm business inventories is modelled as the percentage change in the stock. This is a function of a distributed lag on the percentage change in the sales of private business and the four quarter percentage change in the ratio of corporate profits before tax to private business product.

Although the CHASE model has a Cobb-Douglas production function, factor demands are not modelled consistently with the production function. There is no hierarchy of factor demand response to a demand shock. There are no mechanisms to make sure that the economy operates on the production function in the long run. The model is mainly demand driven.

### 3.4.5 DRI

The main output concept in the DRI model is real GNP. Potential output is defined as full employment GNP which is generated by a Cobb-Douglas production function relating real GNP to labour, capital and energy inputs. Potential GNP is thus expressed as a function of the full-employment labour force (contribution = .637), the fully utilized capital stock at the peak level of capacity utilization in manufacturing (contribution = .296), an implicit energy demand term based on relative energy prices (contribution = .0677), and a time trend for total factor productivity. The function can be summarized:

$$\text{GNP71FE} = f(\text{EFE}, "K", "ENERGY", \text{TIME})$$

where GNP71FE is full employment GNP, EFE is full-employment employment, calculated using the full employment labour force and unemployment rates, "K" is fully utilized capital stock equal to the peak level of capacity utilization in manufacturing times the capital stock, "ENERGY" is a weighted sum of the price of natural gas, petroleum and coal, and electricity to the GNP deflator, and TIME is the time trend for total factor productivity.

There is no requirement that supply as defined by the production function and demand be equilibrated in the long-run.

The investment equations in the DRI model are for investment in non-residential construction and machinery and equipment excluding pipelines, electricity and mining. The equations are characterized in the model documentation as reflecting a "cash flow-augmented neoclassical stock-adjustment model embodying a replacement investment hypothesis". Investment is specified to depend on the gap between the desired and actual capital stock and on the depreciation of the capital stock. The desired capital stock is specified to be a function of real final sales, the user cost of capital, and real cash flow.

The equation for non-residential construction (ICERNE71) can be summarized as follows:

$$\text{ICERNE71} = f(\text{SF71}(-i), \text{PGNP71}/\text{ICERCOST}(-j), \text{CASH71}(-k),$$

$$\text{KNCERNE71}(-l), d * \text{KNCERNE71}(-l), \text{DMYEXPO}, \text{DMYOLYM})$$

where  $i = 1,16$ ;  $j = 4,11$ ;  $k = 1,8$ ;  $d = .021799$ , and SF71 is real final sales, PGNP71 is the GNP deflator, ICERCOST is the user cost for non-residential construction, CASH71 is real cash flow, KNCERNE71 is the stock of non-residential construction, and DMYEXPO and DMYOLYM are dummy variables for Expo 67 and the 1976 Olympics.

The equation for machinery and equipment (IPDENE71) can be similarly summarized:

$$\text{IPDENE71} = f(\text{SF71}(-i), \text{PGNP71}/\text{IPDECOST}(-j), \text{CASH71}(-k),$$

$$\text{KNPDENE71}(-l), d * \text{KNPDENE71}(-l), \text{DMEXPO})$$

where  $i = 1,12$ ;  $j = 4,11$ ;  $k = 1,8$ ;  $d = .0844909$ , and  
 IPDECOST is the user cost for machinery and equipment,  
 KNPDENE71 is the stock of machinery and equipment, and  
 all other variables are as defined above.

The user cost variables in the DRI model for both non-residential construction and machinery and equipment are weighted sums of user costs calculated for manufacturing and non-manufacturing. These user costs incorporate manufacturing, corporate tax rate, accelerated write-offs, investment tax credit rates and bases, and the present value of allowable tax depreciation for both non-residential construction and machinery and equipment. The interest rate utilized is the nominal McLeod, Young, Weir long-term industrial bond rate.

Total employment (E) in the DRI model is specified as the following function of real GNP and of the real wage rate relative to aggregate productivity.

$$E = -5022 + .0340972 * GNP71 + .0227315 * GNP71(-1) \\ + .0113657 * GNP71(-2) + JW((PGNP71/WRATE)/(GNP71/E))$$

where GNP71 is real GNP, PGNP71 is the price deflator for GNP,  
 and WRATE is the wage rate.

The sum of the lag weights on the real wage to productivity term is 36.894.

The equation in the DRI model for the change in non-farm business inventories incorporates both planned and unplanned behaviour. The planned component of inventory change is explained by a stock adjustment model which relates the change in inventories to the gap between the desired and actual stock. The desired stock depends on the level of final sales of goods. The unplanned component of inventory demand is modelled as the difference between expected and actual sales. Expected sales are calculated by scaling up inventories four quarters ago by the four quarter rate of change in the last quarter. The equation can be summarized as:

$$INVEAF71CH = f(INVEAF71, "SFG71", SFG71(-4) \\ * ((SFG71(-1)/SFG71(-5))**.95) - SFG71)$$

where INVEAF71CH is change in real non-farm inventories,  
 INVEAF71 is the opening stock of real inventories, and  
 "SFG71" is the real value of final sales of goods.

Factor demands are not modelled consistently with the aggregate production function which, although it is Cobb-Douglas, has three factors of production and a time trend. The investment equations are said to be consistent with a Cobb-Douglas production function but are in fact quite



eclectic. The ratio of the user cost to the output price is not divided into sales as required by theory. Instead the output and relative price terms are separately included in the equation with different lags. No constraints are imposed to ensure that the sum of the lagged coefficients are the same for output and relative prices as is required by Cobb-Douglas production theory. Also according to the production function there is only one type of capital, but there are two investment equations. The employment equation is also not necessarily consistent with the aggregate production function or the investment equations.

There are no strong supply sector mechanisms to make sure that the economy operates on the production function in the long-run. The only mechanism creating pressures in this direction is the inclusion of capacity utilization, defined as the ratio of actual output to full employment, in the industry selling price equations. This is not a very strong link.

### **3.4.6 FOCUS**

The principal output variable in the FOCUS model is real private domestic product. A production function based on this series yields an independent measure of profit maximizing supply. This production function is unique. It is based on the assumption that the private sector of the economy is made up of  $N$  almost identical plants, and that there is an optimal plant size. Within any given plant production follows a Cobb-Douglas production function with labour and machinery and equipment as factor inputs.

Investment in non-residential construction is associated with the time rate of change in the number of firms. It is stimulated by a positive differential between the rate of return on structures and the real after-tax rate of interest on government bonds. Labour, and machinery and equipment are considered to be partly overhead and partly production. The overhead part is related to the stock of non-residential structures (the number of firms). The production parts are substitutes in the production process based on relative factor costs. Factor demands are modelled consistently based on the underlying production technology.

The production function in FOCUS is:

$$\begin{aligned}
 & (1) \\
 \text{GDPP71S} = & \text{EXP}(.0572801 + .00451935 * \text{DMYTIME} + .301225 * \text{DMYPROD} \\
 & - .00334127 * \text{DMYPROD} * \text{DMYTIME}) \\
 & (2) \\
 & * ((\text{KICER71} * \text{UCAPM} + \text{ICER71} - .0125 * \text{KICER71}) \\
 & * (\text{UCAPM}^{**-.527332})^{**}(1.0 - .370096 - .0483093) \\
 & (3) \\
 & * ((\text{EPRIV} - \text{EOH}) * ((\text{EPROD}/\text{EPROD}^{*})^{**}1.13535))^{**}.370096 \\
 & (4) \\
 & * (((\text{KIPDE71} * \text{UCAPM} + \text{IPDE71} - .0357 * \text{KIPDE71}) - \text{KPOHU}) \\
 & * (\text{UCAPM}^{**-.527332})^{**}.0483093
 \end{aligned}$$

where GDPP71S is real private domestic supply, DMYTIME is a time trend, DMYPROD is a dummy for a shift in the trend growth of productivity, KICER71 is the stock of non-residential construction,UCAPM is the rate of utilization, ICER71 is the investment in non-residential construction, EPRIV is private employment, EOH is overhead employment, EPROD/EPROD\* is the ratio of actual to desired production employment - a measure of labour hoarding, KIPDE71 is the stock of machinery and equipment, IPDE71 is investment in machinery and equipment, and KPOHU is overhead machinery and equipment.

The first term captures total factor productivity allowing for a gradual productivity slowdown due to the oil price shock of late 1973. The second term represents the contribution of non-residential construction; the third production employment. For the capital stock variables an adjustment is made for the degree of utilization. Utilization in FOCUS is a function of the profit rate. The rationale for this is that only at higher profit rates does it pay to utilize the older and less efficient capital stock. For production labour an adjustment is made for labour hoarding.

A potential GNP series is also derived in FOCUS using the production function by substituting private sector employment at the natural rate and capacity utilization at the natural rate into the production function.

Real private domestic supply is very important in the flexible price version of FOCUS where the price level is set to equilibrate aggregate supply and aggregate demand. In the fixed price version supply and demand can diverge for extended periods. However, since the factor

demands are based on the same production technology, supply should exhibit a tendency to come back on track with demand.

The FOCUS equation for non-residential investment (ICER71) is:

$$\begin{aligned}
 \text{ICER71} = & \text{(1)} \quad \text{(2)} \\
 & .0125 * \text{KICER71} + (\text{KICER} + (.75 * \text{RE71} + .25 * \text{J8W}(\text{RE71}(-1)))) \\
 & \text{(3)} \\
 & * (.0042353 + .001 * 4.99378 * (.260716 * \text{RRDIF})) \\
 & + (1.0 - .260716) * \text{J8W}(\text{RRDIF}(-1)) + 302.224 * \text{DMY75}
 \end{aligned}$$

where KICER is the stock of non-residential construction, RE71 is an approximation to real retained earnings, RRDIF is the difference between the real after tax rate of return on non-residential structures and the real after-tax rate of return on short-term government bonds, and DMY75 is a dummy variable equal to 1 in 1975.

The first term represents replacement investment on the existing stock of capital. The second term is a scaling factor for the real after-tax rate of return differential (term (3)). If retained earnings are high, investment responds more to a given interest differential. The real after-tax rate of return on non-residential construction is a complicated construction which is computed as the ratio of net quarterly earnings from new investment to actual costs, assuming that the increase in the capital stock from new investment is maintained in perpetuity. It takes into account corporate income tax and tax savings from capital consumption allowances.

Machinery and equipment in FOCUS is treated partially as overhead and related to the stock of non-residential construction and partially as a variable factor of production which can be substituted for labour depending on the ratio of the user cost of capital to the price of output. This is consistent with the FOCUS assumption that the economy is comprised of N optimum size plants (related to the stock of non-residential construction) and the assumption that the technology for each plant is Cobb-Douglas allowing substitution of machinery and equipment for labour.

The equation for investment in machinery and equipment (IPDE71) is:

$$\begin{aligned}
 \text{IPDE71} = & \text{(1)} \\
 & (.371195 + .00115729 * \text{DMYTIME}) * (.9875 * (\text{KICER71} + \text{ICER71})) \\
 & \text{(2)} \\
 & + \text{W1} * \text{KPPROD} * \text{W2} * (\text{KPGAMMA} * \text{KPPROD} * + (1.0 - \text{KPGAMMA})) \\
 & \text{(3)} \quad \text{(4)} \\
 & * \text{KPPROD} - \text{KIPDE71} + .0357 * \text{KIPDE71}
 \end{aligned}$$

where DMYTIME is a time trend, KICER71 is the stock of non-residential construction, W1 and W2 are weights based on the proportion of new and older structures in the capital stock, KPPROD\* is desired production (variable) machinery and equipment at the end of the period, KPGAMMA is a linear function of the ratio of retained earnings to the stock of capital, KPROD is actual production machinery and equipment at the end of the period, and KIPDE71 is the stock of machinery and equipment.

Term (1) represents the overhead stock of machinery and equipment which is explained by a time trend on the stock of non-residential construction. Term (2) measures the production (variable) machinery and equipment stock as a function of the desired stock and a retained earnings ratio which influences the speed with which the actual stock approaches the desired.

The

desired stock is derived from a Cobb-Douglas production function and is based on a weighted average of current and past private GDP in current dollars divided by the user cost. The user cost variable is a standard Jorgenson user cost with a real interest rate and with the full complement of tax variables including an option to index capital cost allowances.

Term (3) is the actual stock of machinery and equipment. Term (1) and (2) for the desired stock minus term (3) for the actual stock determine the gap between desired and actual stock of machinery and equipment. Term (4) captures replacement demand related to the capital stock.

Employment in the private sector is also divided into overhead and production employment. Overhead employment is related to the utilized stock of non-residential construction, proxying the number of firms. Production employment is based on a lagged adjustment to the desired (cost minimizing) production employment.

The private sector employment (EPRIV) equation is:

$$\begin{aligned}
 & (1) \\
 \text{EPRIV} = & ((.0144 - .00004287 * \text{DMYTIME} + .00013963 * \text{UCAPM}) \\
 & * (.9875 * \text{KICER71} + \text{ICER71}) \\
 & (2) \\
 & + (\text{W1} * \text{EPROD}^* + \text{W2} * (.589156 * \text{EPROD}^* + (1.0 - .589156 \\
 & * \text{EPROD}(-1)))
 \end{aligned}$$

where DMYTIME is a time trend, UCAPM is the capacity utilization rate, KICER71 is the stock of non-residential construction, W1 and W2 are weights based on the proportions of new and older structures in the capital stocks, EPROD\* is desired (cost minimizing) production employment, and EPROD is actual production employment.

The first term represents overhead employment which is tied to the stock of utilized non-residential structures. The second term reflects lagged adjustment to desired production employment. Desired production employment consistent with the Cobb-Douglas production technology, is a lagged function of private GDP at factor cost in current dollars divided by a labour cost variable incorporating the cost of employee contributions to unemployment insurance and the Canada Pension Plan.

The change in real business non-farm inventories in FOCUS is explained as being part anticipated and part unanticipated. The anticipated portion is modelled as the gap between the desired and actual stock of inventories with the desired stock being modelled as a declining time trend function of expected sales. Expected sales is proxied as a simple average of final sales of goods in the current and in the previous periods. The unintended component of inventory change is a function of the gap between actual and expected sales.

The equation for the change in non-farm inventories (INVEAF71CH) in the FOCUS model is:

$$\begin{aligned}
 \text{INVEAF71CH} = & (-5735.26 + (.354552 - \text{DMYTIME}) * (.5 * (\text{FSGOODS71} \\
 & + \text{FSGOODS71}(-1)))) - .149711 * \text{KINVEAF71} - .239154 \\
 & * (\text{FSGOODS71} - .5 * (\text{FSGOODS71} + \text{FSGOODS71}(-1)))) \\
 & + .568225 * (\text{INVEAF71CH}(-1) + 5735.26 - (.354552 \\
 & - .0016282 * \text{DMYTIME}(-1)) * (.5 * (\text{FSGOODS71}(-1) \\
 & + \text{FSGOODS71}(-2)))) + .149711 * \text{KINVEAF71}(-1) + .239154 \\
 & * (\text{FSGOODS71}(-1) - .5 * (\text{FSGOODS71}(-1) + \text{FSGOODS71}(-2))))
 \end{aligned}$$

where DMYTIME is a time trend, FSGOODS71 is real final sales of goods, and KINVEAF71 is the stock of non-farm inventories.

### 3.4.7 MTFM

Output in MTFM is disaggregated into 40 industries. These include: Agriculture; other primary (6 categories); manufacturing (20 categories); construction; commercial services (12 categories); and public administration and defence. The output equations are based on the input/output approach and are computed as identities in the model.

MTFM does not contain industry production functions. Potential output is not defined. The model does not have a fully articulated supply sector.

Non-residential investment in MTFM is broken into machinery and equipment and non-residential construction for nine industrial categories. The industrial categories include: agriculture; other primary; manufacturing; construction; commercial services; and non-commercial. Petroleum and gas mining, petroleum and coal products, and electric power, pipelines and gas distribution are split off into separate energy investment categories.

The investment functions are based on the neoclassical theory of investment behaviour. Some rely on profit maximization, others on cost minimization. The specifications incorporate changes in the desired stock with a lagged capital stock included to reflect replacement investment. The desired capital, following Jorgenson, is a function of industry output and the ratio of the wage rate or output price to the user cost of capital. The user cost is constructed using an industry specific real cost of capital variable that includes a weighted average of the tax adjusted rate on industrial bonds, the TSE 300 earnings- price ratio, and the expected rate of consumer price inflation. It also incorporates industry specific corporate tax rates, effective investment tax credits, and capital consumption allowances.

Employment in MTFM is broken down into 6 industry and 3 age groups for men and women. The industries considered are: agriculture; other primary; manufacturing; construction; commercial services; non-commercial services; and public administration and defence. The industry employment equations are inverted value-added Cobb-Douglas production functions, which specify employment in person-hours as a function of industry output, the capital stock, and trend productivity. The equations are partial adjustment. Person-hours are converted into employment by dividing by average weekly hours. The percentage change in industry output, industry trend labour productivity and permanent income are the main determinants of average weekly hours.

The equation for change in non-farm inventories (IIBNFK) is:

$$\begin{aligned}
 & \text{(1)} \\
 \text{IIBNFK} = & 1944.32 + (-.04685 - .00190 * (\text{RCP90} - \text{J3A}(\text{J1P}(\text{PSALES})))) \\
 & + .63811 * \text{RKIIBSAT} * \text{J1L}(\text{J2A}(\text{SALESK})) \\
 & \text{(2)} \\
 & - .16073 * (\text{SALESK} - \text{J1L}(\text{J2A}(\text{SALESK}))) \\
 & \text{(3)} \\
 & - .49312 * \text{J1L}(\text{KIIBNF})
 \end{aligned}$$

where RCP90 is the 90 day commercial paper rate, PSALES is the price index for sales, PKIIBSAT is the trend stock-to-sales ratio of business non-farm inventories, SALESK is real sales, and KIIBNF is the stock of non-farm inventories.

The change in non-farm business inventories is a function of the gap between the desired and actual inventory stock (term (1) minus term (3)) and the difference between actual and expected sales (term (2)). The desired stock is defined as the product of a desired stock-to-sales ratio and expected sales. The desired stock-to-sales ratio depends on the real interest rate and the trend stock to sales ratio. Expected sales are based on past average sales.

The modelling of factor demands in MTFM is not consistent with any underlying production function either in the aggregate or by industry. The level of disaggregation is not the same for output, investment and employment. The parameters of the implicit Cobb-Douglas production functions utilized in the investment and employment demand equations are not constrained to be the same. Factor demands are only loosely related to the extent that capital stock is included in the employment equations. Since there is no underlying consistent concept of supply, the economy is not required to operate on a production surface in the long-run.

### 3.4.8 QFS

Aggregate supply in the QFS model is patterned on that in RDX2. However, there are a number of potential inconsistencies and different concepts of output in the model which can be traced to its origin as an assemblage of sectoral models produced by sectoral specialists. The output concepts include RDP in manufacturing and in service industries excluding public administration, manufacturing gross domestic product, total sales of goods, and nominal and real gross private business product.

The supply of output for real gross private business product is specified as a four factor Cobb-Douglas production function with the stocks of machinery and equipment and non-residential construction, energy and labour as inputs. The production function is:

$$\begin{aligned} \text{LOG(UGPPS)} = & (\text{ABCL\_AVG} + .0025 * \text{TY1} - .0005 * \text{TY2}) + .15 * \text{LOG(KME71)} \\ & + .09 * \text{LOG(KNRC71)} - .05 * \text{LOG(PCPIENR/PGPPI)} \\ & + .71 * \text{LOG(LFEXAPA)} * (1/.95) \end{aligned}$$

where UGPPS is the supply of real gross private business product, ABCL\_AVG is ratio of average output to production function output, TY1 is a quarterly time trend up to 73Q4, TY2 is a quarterly time trend starting in 74Q1, KME71 is the stock of machinery and equipment, KNRC71 is the stock of non-residential construction, PCPIENR is the CPI energy price index, PGPPI is the price index for gross private business product, and LFEXAPA is employment excluding public administration.

It is useful to note in the QFS model that the utilization of two time trends introduces an undesirable discontinuity in total factor productivity. Moreover, there is no requirement that supply as defined by the production function and demand be equal in the long-run.

Potential output is defined using the same production function with employment at the NAIRU substituted for actual employment. The QFS capacity utilization variable based on potential output is defined as a four quarter moving average of the ratio of actual output adjusted for unintended inventory change relative to potential. It appears in some of the price equations. Others contain the ratio of actual to desired inventories. Another capacity utilization rate is that for manufacturing. In inverse form, it is used in the import equations.

Investment in QFS is disaggregated into machinery and equipment and non-residential construction and into total and manufacturing and into energy and non-energy. The equations are based on the Jorgenson model. The interest rate utilized in the rental cost is the nominal rate on long-term industrial bonds. The form of the equations is straightforward. The equation for machinery and equipment (IME71) is:

$$\begin{aligned} \text{IME71} = & .00132 * \text{JW}(\text{J1D}(\text{J3L}(\text{PGNE} * \text{GNE71}/\text{RRCME}))) \\ & + .03378 * \text{KME71} \end{aligned}$$



where PGNE is the GNE deflator, GNE71 is real GNE, RRCME is the user cost of machinery and equipment, and KME71 is the stock of machinery and equipment.

The lag term is 23 quarters long and the sum of the lag weights is .207

The equation for non-residential construction (INRC71) is:

$$\text{INRC71} = -.000163 * \text{JW}(\text{J1D}(\text{J3L}(\text{PGNE} * \text{GNE71}/\text{RRCNR}))) \\ + .01972 * \text{KNRC71}$$

where RRCNR is the user cost of non-residential construction, and KNRC71 is the stock of non-residential construction.

The lag term is 27 quarters long and the sum of the lag weights is .060.

The user cost incorporates corporate tax rates, the investment tax credit, and capital consumption allowances for the manufacturing sector and in total for both types of capital investment.

The QFS equation for employment excluding agriculture and public administration (LFEXAPA) is:

$$\text{LOG}(\text{LFEXAPA}) = 1.32572 + .04084 * \text{LOG}(\text{UGPPA}/\text{J4A}(\text{UGPPD})) \\ + \text{JW}(\text{LOG}(\text{WAYAI}/\text{PGPPI})) + \text{JW}(\text{UGPPA})$$

where UGPPA is private business product adjusted for unintended change in inventories, UGPPD is potential output, WAYAI is the average wage, and PGPPI is the deflator for private business product.

The sum of the lag coefficients on the real wage is -.568. The sum of the lag coefficients on output is .908.

The equation for the stock of inventories incorporates both planned and unplanned holdings. Planned inventories are based on a partial adjustment to desired stocks which are a function of expected sales and holding costs. Holding costs are specified as the real after-tax rate of return as measured by the prime lending rate. Unplanned inventory holdings result from deviations of expected from actual sales.

There is no consistent production function underlying the factor demand equations. The investment equations are based on a Cobb-Douglas production function while the employment equations are not based on any particular production technology. There is no constraint in QFS that requires the economy to operate on its supply function. Since the factor demands are not derived from the supply function, there is no self-equilibrating mechanism to bring supply into line with demand in the long-run.

### 3.4.9 RDXF

The concept of output that is central in RDXF is non-agricultural private domestic business product at factor cost excluding paid and imputed rent (UGPP). Supply of output is based on a three factor Cobb-Douglas production function of the following form:

$$UGPPS = (J1L(KMEXE)**.18) * (J1L(KNRCXE)**.16) * ((NIC * HAWMM * 52)**.66) * ETFP$$

where UGPPS is the supply of output, KMEXE is the stock of machinery and equipment excluding energy, KNRCXE is the stock of non-residential construction excluding energy, NIC is employment in the industrial composite, HAWMM is average weekly hours worked in mining and manufacturing, and ETFP is total factor productivity.

Aggregate supply does not play much of a role in RDXF and is primarily a display variable. The economy is not required to operate on its production function in the long-run.

The potential output concept in RDXF is normal output which is defined as the output from the Cobb-Douglas production technology using the existing capital stocks, trend hours and private sector employment at trend output. Private sector output at trend output is calculated by multiplying the labour force by the employment rate at trend output (one minus the natural rate of unemployment) and subtracting trend employment in the government and non-commercial sector.

Actual output divided by normal output is the RDXF measure of capacity. It appears in some of the price and some of the import equations.

The investment equations in RDXF are for non-energy investment in machinery and equipment and non-residential construction. Energy investment in these two categories is exogenous. The original formulation of the RDXF equations was based on the Jorgenson model assuming profit maximizing behaviour with investment being a function of the gap between the desired and actual capital stocks. The desired stock was defined as the share of capital multiplied by output times price divided by the user cost. In the latest version, the desired stock is a function of output

times

the user cost divided by the wage rate. Such a specification resembles one based on cost minimization but is not rigorously derived as the wage is not in efficiency units and the ratio of the relative prices of the two types of capital is omitted.

The RDXF equation for investment in machinery and equipment is:

$$\begin{aligned} (\text{IME} - \text{IMENRG})/\text{J1L}(\text{KMEXE}) &= -.60696 + .34090 * \text{J1L}(\text{KMEXE}) \\ &+ \text{J10W}(\text{LOG}(\text{UGPP})) + \text{J4W}(\text{LOG}(\text{RCME}/ \\ &((52 * \text{WNICIR})/(52 * \text{HAWMM})))) \end{aligned}$$

where IME is investment in machinery and equipment, IMENRG is energy investment in machinery and equipment, KMEXE is the stock of non-energy machinery and equipment, UGPP is private business product, RCME is the user cost of machinery and equipment, WNICIR is average weekly wages grossed up for special payments minus losses due to strikes, and HAWMM is average weekly hours in mining and manufacturing.

The sum of the lag weights on UGPP is .40435. The sum of the lag weights on the relative cost term is -.02914.

The RDXF equation for non-residential construction is:

$$\begin{aligned} (\text{INRC} - \text{INRCE})/(\text{J1L}(\text{KNRCXE})) &= 1.54174 + .53823 * \text{J1L}(\text{KNRCXE}) \\ &+ \text{J13W}(\text{LOG}(\text{UGPP})) + \text{J6W}(\text{J1L}(\text{RCNR}/((52 * \text{WNICIR} * \text{HAWMM})))) \end{aligned}$$

where INRC is investment in non-residential construction, INRCE is energy investment in non-residential construction, KNRCXE is the stock of non-energy investment in residential construction, RCNR is the user cost of capital in non-residential construction, and the other variables are as defined above.

The sum of the lag weights on UGPP is .40215. The sum of the lag weights on the relative cost term is -.03211.

The user cost in RDXF includes the real supply price of capital, the weighted corporate income tax rate, the weighted investment tax credit rate, and the present value of capital consumption allowances for both machinery and equipment and non-residential construction.

The main employment equation in RDXF is for employment in the industrial composite. The specification is eclectic incorporating a lagged adjustment to desired employment calculated by inverting the production function utilizing average output and actual capital stocks. It also includes the real wage and output as additional explanatory variables. These are the factors that influence desired employment after the capital stock has adjusted to desired levels. The equation is:

$$\begin{aligned} \text{LOG(NIC)} = & -1.46676 + .09026 * \text{LOG(NICD)} + .59446 * \text{J1L(NIC)} \\ & - .14268 * \text{J4A}(\text{LOG(WNIC} * (1 + \text{RSPIC)/PGPP}) \\ & + .24630 * \text{LOG(UGPP)} \end{aligned}$$

where NIC is employment in the industrial composite, NICD is desired employment calculated by inverting the production function, WNIC is the wage rate for the industrial composite, RSPIC is supplementary labour income not dependent on government policy as a proportion of wage income, PGPP is the price deflator for private business product, and UGPP is private business product.

The hours worked (HAWMM) equation is:

$$\text{LOG(HAWMM)} = 3.77948 + .00108 * \text{QTIME} + .24191 * \text{LOG(UGPP/UGPPS)}$$

where HAWMM is average weekly hours worked in mining and manufacturing, QTIME is a time trend, and UGPP/UGPPS is the RDXF aggregate measure of capacity utilization.

The equation for the change in non-farm business inventories in RDXF reflects both planned and unplanned inventory holdings. The planned change in inventories depends on the gap between the actual and desired stock. The unplanned change results from the difference between expected and actual sales.

The RDXF equation for the change in non-farm business inventories (IIB) is:

$$\begin{aligned} \text{IIB/4} = & 665.1220 + \overset{(1)}{.01704} * \text{USALE} + \overset{(2)}{.05219} * (\text{USALE} - \text{USALG}) \\ & - \overset{(3)}{72.73153} * \text{RCCKIB} - \overset{(4)}{.07232} * \text{J1L(KIB)} + 251.8206 \\ & * \text{QSTRK} - 355.9405 * \text{QREB75} - 181.9351 * \text{QREB78} \end{aligned}$$

where USALE is expected real sales of goods calculated by scaling up last period's sales by the four-quarter rate of change in the last period, USALG is real sales, RCCKIB is the after-tax capital cost of holding inventories, KIB is the stock of inventories, and QSTRK, QREB75, and QREB78 are dummy variables reflecting various events such as strikes and rebates affecting inventories.

Term (1) is expected sales and term (3) is the after-tax cost of holding inventories. The latter is defined as the nominal prime rate minus the percentage change in the price of goods times one minus the corporate tax rate. These two terms are the determinants of desired inventory stocks which is contrasted with the actual stock, term (4). Term (2) captures the unintended component of inventory change as the gap between expected and actual sales.

Factor demands are not estimated consistently in RDXF based on an underlying production technology. There is no guarantee that the economy will operate on its supply curve in the long-run. Short-term productivity responses persist for many years (the documentation suggests they are still significant after 8 years).

### **3.4.10 MACE**

Output in the MACE model is defined as gross output at factor cost of the the non-energy sector. Production technology is modelled as a vintage constant-elasticity-of-substitution (CES) bundling of energy and capital nested within a Cobb-Douglas function for gross output based on the bundle and efficiency units of labour. The vintage feature results from scaling new investment (and existing capital becoming malleable) by the CES function of the capital costs relative to energy, and accumulating allowing for depreciation and malleability.

The concept corresponding to potential output in MACE is vintage-based synthetic supply based on the production function utilizing a two year average of the vintage capital-energy bundle and actual non-energy employment in efficiency units (scaled by labour supply). Vintage based synthetic supply has been specified to equal actual output on average over the sample period.

Factor demands in MACE are based on cost minimizing behaviour by firms. Business fixed non-energy investment demand is specified as a proportion of the capital stock and exhibits lagged adjustment to the gap between the desired and actual non-energy capital stock. Since the rental price of capital is taken to be a constant, an additional variable is included to allow financial conditions to influence the rate at which investment takes place. This variable is the ratio of

current unit operating costs including interest to the current output price (or 1 minus the rate of abnormal profits).

The form of the investment equation is:

$$\begin{aligned} \text{Ine}/\text{J2A}(\text{Kne}) = & .40545 * \text{Ine}(-1)/\text{J2A}(\text{Kne}) + .041529 \\ & * (\text{K}^* - \text{J2A}(\text{Kne}))/\text{J2A}(\text{Kne}) - .055326 \\ & * \text{Cq} + .10365 \end{aligned}$$

where Ine is non-energy investment, Kne is the non-energy capital stock, K\* is the desired capital stock, and Cq is the ratio of current unit operating costs to the current output price.

The optimal capital stock is defined as the desired level of future output for investment demand times the optimal capital-output ratio derived from the production function given factor prices for capital, labour and energy. The desired level of output for investment demand is calculated for two years in the future by applying the growth rate over the previous two years to aggregate demand adjusted for unintended inventory accumulation and for the differential between non-energy imports at actual and full capacity levels of output.

Tax variables are not explicitly incorporated in the price of capital services. After-tax variables are used.

Employment demand in the non-energy sector is specified as a Koyck lag on desired employment. Desired employment is defined to be the desired level of future output two years in the future calculated similarly to that for investment demand times the labour-output ratio determined by solving the vintage-based synthetic supply equation given the actual vintage capital-energy bundle. Employment demand is thus based on expected output and the production function and not directly affected by relative factor prices.

MACE has an output equation which determines the level of utilization. The economy is not constrained to operate on the production function in the short-run. The level of utilization responds positively to the ratio of absorption plus exports to supply and to the desired stock of inventories relative to the actual. It responds negatively to average unit costs.

Inventory change is expressed as the differential between output and sales. It plays an important role as a buffer in MACE.

The hierarchy of adjustment is utilization rate, labour and capital in that order.

Factor demands are modelled consistently.

### 3.4.11 SAM

The key output measure in SAM is for private non-energy output. The production technology is a constant-elasticity-of-substitution (CES) structure of capital and energy bundled within another CES. The production function is a stochastic equation. Its form is:

$$UGPC = a_{10} * (a_5 * a_8 * (a_4 * (CAPU * J2A(KCT)))^{a_6} \\ + (1 - a_4) * ENC^{a_6})^{(a_7/a_6)} + (1 - a_5) \\ * (LC * PRODL)^{a_7})^{(1/a_7)}$$

where UGPC is real private non-energy output, CAPU is capacity utilization of capital, KCT is the actual stock of non-energy capital, ENC is energy usage. LC is employment in the non-energy private sector, and PRODL is a labour productivity index representing technical change.

Steady state or potential output is calculated in SAM using the steady state energy price, the steady state cost of capital in efficiency units (based on steady state capital cost equal to the rate of growth of population, plus the rate of growth of productivity and the rate of time preference), the steady state wage in efficiency units, and the assumption that in the steady state the labour market clears and all of the labour offered is employed (up to the natural rate of unemployment). The factor prices and static firm optimization rules yield the optimal output-labour ratio. Multiplying this by the steady state labour force in efficiency units gives steady state or potential output.

The demand for capital in SAM, of which there is only one kind, is expressed as a stock adjustment function of the desired capital stock which depends on steady state output and capacity utilization, and the relative factor prices. It is based on the optimization condition derived from the production function. Disequilibrium variables are also added for the deviation of current capital cost from its equilibrium value, the market valuation of capital relative to its equilibrium valuation (Tobin's Q), the ratio of the steady state output price to its actual level, and the ratio of normal sales to steady state output. Tax variables impact on investment in SAM through a variable for the combined effect in the steady state of investment incentives and taxes on capital costs which is entered multiplicatively in the equation for the steady state cost of

capital.

The demand for labour in SAM is also one of a lagged adjustment based on the static optimization condition from the production function. Disequilibrium effects are introduced through the inclusion of the ratio of the steady state to the actual wage, the ratio of desired to actual inventory stocks, and the gap between actual and steady state capacity utilization. An equation for energy demand based on the same framework is also included in SAM.

The adjustment process in SAM differs from that in the larger quarterly and annual models and is similar to that in MACE. Capacity utilization is explicitly modelled rather than treated as a residual. It is a function of steady state capacity utilization with deviations resulting from the ratio of normal sales to steady state output, the ratio of sales to output which can be supplied at planned levels of utilization, the gap between desired and actual inventory stocks, and profits relative to normal levels.

The residual in the short-run in SAM is inventories. The change in inventories is the difference between output and the sum of consumption, investment, government non-wage spending, and net exports. Since this resolves any differences between output and demand, the economy always operates on the production function as defined above. However, this is a short-run function. In the long-run the binding technology imposes a restriction on the level of capacity utilization, CAPU, that it be at some optimal value, CAPUSS. The short-run movements away from CAPUSS can be thought of as moves off the production function in the usual sense. Inventories thus have a critical short-run buffering role in SAM. In the long-run SAM specifies a desired level for inventories, dependent on potential output and inventory costs. SAM contains feedback mechanisms (through capacity utilization, factor demands, and pricing) to ensure that in the long-run inventory stocks return to desired levels.

The hierarchy of adjustment away from factors of production in SAM to a demand shock involves the greatest short-run variation in capacity utilization and inventory stocks, followed by energy, labour, and finally capital.

Factor demands are modelled consistently.

### **3.5 Income Distribution**

#### **3.5.1 CANDIDE 2.0**

The income side of the CANDIDE model with the exception of the wage bill is not disaggregated by industry. However, the individual components are generally modelled. Capital consumption allowances is determined by a rate base calculation on the capital stock. Income of non-farm



unincorporated business is influenced by the current dollar value of imputed and paid rent and the unemployment rate. Farm income is dependent on farm production and prices. Interest and miscellaneous investment income is determined by an add-up of the components. Once the components of net national income are determined, profits are derived residually. Since profits are the residual and since wages respond with a lag to shocks, profits fluctuate proportionately the most in response to changes in interest rates, inflation and output.

### **3.5.2 TIM**

GDP is derived in TIM by adding to GNP, the residual error of the estimate and subtracting net indirect taxes and subsidies and net foreign receipts. The components of GDP are disaggregated as the sum of factor incomes including labour income, return to capital, capital consumption allowance and income from unincorporated business. The national accounts definitions of incomes are also modelled. As in CANDIDE corporate profits are treated as a residual.

### **3.5.3 RDX2**

Behavioural equations are found in RDX2 for some income components including dividends and net income of non-farm unincorporated business including rent. Interest income, however, is exogenous. The original version of RDX2 had an equation for corporate profits before tax as a function of three explanatory variables approximating charges for the use of capital, sales plus inventory accumulation, and the cost of labour. In the original version the discrepancy between total national accounts expenditure and income was treated as an endogenous variable and included in personal income for simulations. In the latest version of RDX2 corporate profits are a residual.

### **3.5.4 CHASE**

In the CHASE model, except for wages, IVA, and capital consumption allowances, private sector income components are modelled as shares. The wage bill is calculated from wages and employment. The change in capital consumption allowances depends on nominal investment. Corporate profits are the income side residual.

### **3.5.5 DRI**

DRI has equations for interest income, dividends, capital consumption allowances, and corporate profits. Corporate profits as a proportion of GNP are a function of prices, wages as a share of GNP, and capacity utilization. The income side residual in the DRI model is the income of non-farm unincorporated business including rent. It is defined as national income less all other components of income and is included in personal income.

### **3.5.6 FOCUS**

FOCUS has equations for interest income and dividends paid to non-residents. It also has equations for corporate profits, IVA and capital consumption allowances. Corporate profits as a share of total property income are inversely related to the unemployment rate and directly related to the rate of capacity utilization, the ratio of real final goods sales to real private GNP and the ratio of non-energy merchandise exports to private GNP in current prices. The income-side residual in FOCUS as in the DRI model is the income of unincorporated business including rent. This reflects the common descent of the FOCUS and DRI models from the earlier University of Toronto Quarterly Forecasting Model (QFM).

### **3.5.7 MTFM**

MTFM has stochastic equations for many of the private sector income side components in addition to determining the wage bill using industry equations for wages and employment. The components modelled include: business CCA, non-farm unincorporated business, IVA, interest and miscellaneous investment income of persons. The equation for the latter explicitly incorporates financial asset stocks. Corporate profits are determined residually.

### **3.5.8 QFS**

The various components on non-wage net national income are modelled as functions of related GNE and financial variables. Corporate profits are a function of unit labour costs, real final domestic demand, the final domestic demand deflator and real exports. The equation is specified as a first difference in the current value of the LHS and RHS variables divided by the lagged variables. The equation is also corrected for autocorrelation. The income expenditure residual is allocated across components in relation to their respective shares of non-wage net national income.

### **3.5.9 RDXF**

RDXF has stochastic equations for most important private sector income components. These include: accrued net income of farm operators from farm production; miscellaneous personal investment income; net income of non-farm unincorporated business; and capital consumption allowances. The equation for corporate profits is specified as a proportion of GNP and includes asset stocks, interest rates, the exchange rate, and various income and expenditure components. It also includes a variable which is the sum of corporate profits before tax, miscellaneous personal investment income, and the net income of non-farm unincorporated business including rent. This variable, which can also be viewed as the income side residual, is spread amongst its three components in simulations so that the income and expenditure side of the national accounts balance.

### 3.5.10 MACE

MACE only has a very rudimentary income sector. All components are not specified and there is no income side add-up. Items that are treated explicitly are the wage bill and capital consumption allowances. Indirect taxes are also modelled. Corporate profits are not defined in MACE.

### 3.5.11SAM

SAM has a highly aggregated income sector. Income is split between the wage bill, gross profits, payments to energy, and indirect taxes. Business profits are technically a residual and thus reflect disequilibrium factor payments as excess profits. Profits feed through to the asset sector as the return on equity. Because of the highly aggregated nature of the income distribution, the SAM measure of profits is not identical to the national accounts measure of corporate profits, but includes other business income components not treated separately.

## 3.6 Housing

### 3.6.1 CANDIDE 2.0

The CANDIDE 2.0 housing block includes: 1) expenditure on single housing units; 2) expenditure on multiple housing units; 3) expenditure on mobile homes; 4) expenditures on cottages; and 5) expenditures on real estate commissions and; 6) an "other" category.

The key single and multiple housing starts equations are a function of new residential mortgage approvals, demographic factors, the relative advantage of renting versus owning, a proxy for the lagged change in vacancies, and dummy variables for government policies such as winter works programs and the deductibility of capital consumption allowances from other income.

The single housing starts (RSS) equation is:

$$\begin{aligned} \text{RSS} = & \text{(1)} \\ & 128.54600 + .019894 * (\text{FMAP.PNRWRT} + \text{FMAP.CMHC.SD})/\text{PFRCINAB} \\ & \text{(2)} \\ & + .0081887 * (\text{FMAP.PNWRTC}(-1) + \text{FMAP.CMHC.SD}(-1))/\text{PFRCINAB}(-1) \\ & \text{(3)} \\ & + 5.29181 * \text{RDUMWW} \\ & \text{(4)} \\ & + 119.69400 * (\text{PFHOWIX}/\text{PFCSR20}) \\ & \text{(5)} \\ & + .72398 * \text{J1D}(\text{DMPOP30.34} + \text{DFPOP30.34}) \end{aligned}$$

where the first (1) and second (2) terms are the value of private and CMHC mortgage approvals deflated by the residential

construction deflator, RDUMWW is the winter works dummy, the fourth (4) term represents the cost of home ownership relative to renting, and the fifth (5) term is the change in the population between age 30 and 34.

The multiple housing starts (RMS) equation is:

$$\begin{aligned}
 \text{RMS} = & 8.29247 + \overset{(1)}{.023538} * (\text{FMAP.PNWRT} + \text{FMAP.CMHC.MD})/\text{PFCINAB} \\
 & + \overset{(2)}{.0043858} * (\text{FMAP.PNWRT}(-1) + \text{FMAP.CMHC.MD})/\text{PFCINAB}(-1) \\
 & + \overset{(3)}{22.71000} * \text{RDRCCAIP}(-1) \\
 & - \overset{(4)}{.19724} * (\text{RMS}(-2) - \text{J1D}(\text{DMPOP20.24}(-2) + \text{DMPOP25.29}(-2))) \\
 & + \text{DFPOP20.24}(-2) + \text{DFPOP25.29}(-2)) \\
 & + .12700 * \text{J1D}(\text{DMPOP20.24} + \text{DMPOP25.29} + \text{DFPOP20.24} + \text{DFPOP25.29})
 \end{aligned}$$

where terms (1) and (2) are real mortgage approvals, RDRCCAIP is the capital consumption allowance dummy variable, term (4) is the difference between lagged multiple starts and the change in the number of young people age 20 to 29, the latter being a proxy for demographic demand, which is also entered separately as term (5).

Completions and expenditures on residential construction are explained as a lag on single and multiple housing starts. Expenditure also includes real estate commissions, which are explained by a rate-base equation, and an other category, which is influenced by real disposable income and completions.

The major impact of financial variables on housing functions through the mortgage approval-mortgage assets channel. The mortgage approvals of insurance companies, trust and mortgage companies, and chartered banks are determined by their earning assets and interest rates on mortgages and competing assets.

### 3.6.2 TIM

The TIM housing sector explains both single and multiple housing starts. Two simplifying assumptions are made: 1) families prefer single units, and 2) single people live in multiple units.

The equation for single starts (HSS) is:

$$HSS = a_0 + a_1 * (.75 * PFES - PFESRR) * FAMHO + a_2 * REALI$$

where PFES is the percentage of families eligible for single units (this is a function of disposable income per household divided by the average mortgage payment), PFESRR is the percentage of families estimated to be already in singles, FAMHO is family households, and REALI is the real interest rates defined as yield on commercial paper minus the percentage change in the GNE deflator.

The equation for multiple starts (HMS) is:

$$HMS = b_0 + b_1 * ((PFEM - PFEMRR) * FAMHO) + b_2 * J1D(NFHO) + b_3 * (CANPCP - J1P(CSR2OP))$$

where PFEM is the percentage of families eligible for multiple units (this is a five year distributed lag on disposable income per household divided by the average mortgage payment), PFEMRR is the percentage of families estimated to already be residing in multiple units, FAMHO is the number of family households, NFHO is the number of non-family households, CANPCP is the commercial paper rate, and CSR2OP is the price deflator for paid space rent.

The average mortgage payment used in calculating the percentage of families eligible for the units is a function of interest rates, land costs and house costs.

Completions and expenditures are derived as distributed lags on starts. Expenditures also incorporate: expenditures on conversions which are exogenous; alterations and additions, which depend on disposable income and the housing stock; and supplementary costs of new housing which are tied to housing starts.

### 3.6.3 RDX2

The original version of RDX2 explained housing starts in a reduced form equation incorporating real disposable income per household and credit conditions. In the latest version of RDX2 housing starts are tied directly to mortgage loans by a technical relationship to reflect the fact that most residential construction is financed by mortgage loans. The equation for housing starts was specified:

$$\begin{aligned}(14.765 * HSTS + 9.843 * HSTM) = & -211.62 * (1-ZWW) * QC1 \\ & + 91.913 * (1-ZWW) * QC2 + 84.562 * (1-ZWW) * QC3 \\ & + 151.49 * (1-ZWW) + 100.84 * ZWW - 183.36 + ZWW * QC1 \\ & + 20.500 * ZWW * QC2 + 83.284 * ZWW * QC3 \\ & + JW((HAPNRESD + HAPCMHCS + HAPCMHCM + HAPNROT)/PIRC)\end{aligned}$$

where ZWW is a dummy variable for winter works equal to 1 from 63Q3 to 66Q2, QC1, QC2, and QC3 are quarterly dummy variables, HAPNRESD is mortgage approvals for new residential construction by life insurance, trust, and mortgage and loan companies, and chartered banks, HAPCMHCS is direct CMHC loans for singles, HAPCMHCM is direct CMHC loans for multiples, HAPNROT is mortgage approvals by other lenders, and PIRC is the deflator for residential construction.

The constants 14.765 and 9.843 are the average cost for a single and multiple unit under the National Housing Act in 1961. These constants translate starts into dollars. The sum of the weights on mortgage approvals is 1.07184. This is very close to the value of 1 which would occur if all units were totally financed by mortgages.

The housing starts equation was solved for single units and the number of multiple units was simultaneously determined on the basis of an exogenous split.

To understand how interest rates impact on housing starts it is necessary to go beyond the housing start equation into the determination of mortgage interest rates and mortgage approvals. There are two main ways that an increase in interest rates would affect housing starts in RDX2. First, higher interest rates would reduce the total assets of mortgage lenders. Second, as interest rates went up, the gap between the conventional mortgage rate and the average yield on 3 to 5 year government bonds would narrow, thus encouraging trust companies and banks to devote a smaller share of their assets to mortgage lending. This would cause housing starts to decrease.

The equations for mortgage approvals can be viewed as supply of funds equations. The key demand equation is for the conventional mortgage rate to which the bank and NHA mortgage rates are tied. The conventional mortgage rate responds with a Koyck lag negatively to real housing approvals per household and the stock of housing per household and positively to the long-term government bond rate and permanent income.

Expenditure on residential construction is composed of new housing expenditures, alterations, and supplementary housing stock costs. Expenditure on new housing depends on lagged housing starts. Other expenditures on residential construction is modelled as a function of the lagged housing stock.

### 3.6.4 CHASE

The CHASE model has one stochastic equation for housing starts. The split between singles and multiples is done using an exogenous ratio.

The key starts equation incorporates both demand and supply elements and is as follows.

$$\begin{aligned} & (1) \\ \text{LOG}(\text{HSCA}/\text{NPOPCA}) &= 5.78950 + 2.17797 * \text{LOG}(\text{YRPPCA}/\text{NPOPCA}) \\ & (2) \\ & - 3.45260 * \text{LOG}(\text{KRES DCA}/\text{NPOPCA}) \\ & (3) \\ & - .06065 * \text{RMTCCA} + .17977 * \text{DUMMY} \\ & (4) \\ & + .34756 * \text{LOG}(\text{PMLSCA}/\text{PHCCA}) \end{aligned}$$

where HSCA is total housing starts, YRPPCA is real effective purchasing power in 1971 dollars, NPOPCA is the non-institutional population 15 years and over, KRES DCA is the stock of houses in 1971 dollars, RMTCA is the conventional mortgage rate, DUMMY is a dummy variable equal to 1 in 78Q1 and -1 in 78Q2, PMLSCA is the

price of multiple listing service houses, and PHCCA is construction cost per unit for single detached units.

Term (1) captures the impact of real purchasing power per capita on housing demand, Term (3) indicates the negative impact of mortgage interest rates on housing demand. Terms (1) and (3) together could be viewed as giving the desired stock of housing per capita. Term (2) reflects the housing stock per capita. The difference between the sum of terms (1) and (3) and term (2) could be interpreted as the gap between actual and desired housing demand. Term (4) introduces a supply element. The ratio of the multiple listing price to construction costs is an indicator of the profitability of housing construction to developers.

The key price equation for housing is that for multiple listing service houses. It is a demand price modelled by inverting a demand for housing function. Its specific functional form is:

$$\begin{aligned} \text{LOG(PMLSCA/PCICA)} &= \overset{(1)}{-.09271} + 1.03247 * \text{LOG(YRPCCA/NPOPCA)} \\ &+ \overset{(2)}{.59892} * \text{LOG(PMLSCA(-1)/PCICA(-1))} \\ &- \overset{(3)}{.0010763} * (\text{RMTCA} - \text{J1P(PMLSCA(-1))}) \\ &- \overset{(4)}{.61558} * \text{LOG(KRES DCA/NPOPCA)} \end{aligned}$$

where all variables are as defined above with the addition of PCICA which is the consumer price index.

Term (1) is the real effective purchasing power per capita and reflects demand as does term (3) which is the real mortgage rate. Term (4) represents the lagged stock of housing per capita. The difference between the sum of the terms (1) and (3) and term (4) is an indicator of the disequilibrium between demand and supply. Term (2) is the lagged left hand side variable and allows for lagged adjustment.

The other key variable in the housing sector is construction cost per unit for single detached dwellings. It is a function of labour costs, energy prices, and total factor productivity in construction. Construction costs per unit is a supply price that enters in the housing starts equation as the ratio of the demand price to the supply price, measuring the profitability of home building.

Expenditure on residential construction depends on lagged housing starts. An exogenous scaling ratio relating total investment in residential construction to new housing is utilized to allow for other residential construction expenditures.

### 3.6.5 DRI



The main housing start equation in the DRI model is a reduced form for total starts incorporating demand and supply elements. Its general form is:

$$(1)$$

$$\text{HUSTDRI} = f(\text{YD}, \text{RMMTGNS}, \text{PICR71}), \text{KHOUS}(-1), \text{CPI}@@\text{CSRENT}\&/\text{CPI},$$

$$\text{"CREDIT71"}, \text{DMYMURB}, \text{DMYEXP}) * \text{N15}$$

where HUSTDRI is additively adjusted total housing starts, YD is nominal disposable income, RMMTGNS is the mortgage rate, KHOUS is the stock of housing, CPI@@CSRENT&/CPI is the relative rental income as proxied by the consumer price index for rental income to the all items index, "CREDIT71" is the deflated sum of personal savings deposits and non-personal term and notice deposits at chartered banks, DMYMURB captures the impact of MURBs on housing starts, DMYEXP is included to reflect the crowding out impact of Expo 67 on housing, and N15 is the population over 15.

Term (1) represents the burden of homeownership as proxied by a function of disposable income, mortgage rates, and the costs of new housing.

Another equation explains the proportion of multiples to total housing starts (HUST2&%). It is of the form:

$$\text{HUST2}\&\% = f(\text{CPI}@@\text{CSRENT}\&/\text{CPI}, \text{YD71}/\text{N15}, \text{DMYMURB}, \text{DMYMURB814}),$$

$$\text{N15}/(\text{N} - \text{N15}\&))$$

where YD71 is real disposable income, DMYMURB814 allows for the cancellation of MURBs in the November 1981 budget, NR15& is the 15 and over population, and (N - N15&) is the under 15 population.

It is thus a function of the rental income proxy, per capita real disposable income. The function is homogenous of degree one with respect to the ratio of the 15-and-over population to the under-15 population.

Single starts are explained from the housing starts identity. Completions and expenditures on residential construction are lagged functions of housing starts. The equation for expenditures on residential construction also includes a housing stock term to proxy renovation/repair

expenditures and real estate commissions paid on resales of the existing housing stock.

### 3.6.6 FOCUS

The housing sector of the FOCUS model is patterned on that of RDX2. According to the RDX2 specification housing starts are driven by real mortgage approvals. An important difference between FOCUS and RDX2 is that in FOCUS separate equations exist for both single and multiple starts instead of a single equation for weighted starts.

The equation for single housing starts (HUST1NS) is:

$$\begin{aligned} \text{HUSTS1NS} = & 15.7674 + (a_i * \text{constrained seasonal dummies} \\ & * (1 - \text{DUMWORKS}) + (b_i * \text{constrained seasonal dummies} \\ & * \text{DUMWORKS}) - 1.0993 * \text{DUMWORKS} \\ & + \text{JW}((\text{APPLMTGNWRPRIVNS} + \text{APPLMTGNWRCMHCNS})/\text{PICR71VR}) \\ & + \text{JW}(\text{DUMTAXREFORM} * (\text{APPLMTGNWRPRIVNS} \\ & + \text{APPLMTGNWRCMHCNS})/\text{PICR71VR}) \end{aligned}$$

where DUMWORKS is a dummy variable for winter works in the 1960s, APPLMTGNWRPRIVNS is mortgage approvals by private sources such as banks, life insurance companies, trust and mortgage and loan companies and others, APPLMTGNWRCMHCNS is approvals for new construction by CMHC, PIC71VR is the implicit price deflator for residential construction, and DUMTAXREFORM is a dummy variable representing the impact of tax reform and the exception of owner occupied housing from capital gains in increasing the demand for housing.

The equation for multiple units (HUST2&NS) is similarly specified:

$$\begin{aligned} \text{HUSTS2\&NS} = & -6.19528 + (a_i * \text{constrained seasonal dummies} \\ & * (1 - \text{DUMWORKS})) \\ & + (b_i * \text{constrained seasonal dummies} * \text{DUMWORKS}) \\ & - 2.8131 * \text{DUMWORKS} \\ & - \text{JW}((\text{APPLMTGNWRPRIVNS} + \text{APPLMTGNWRCMHCNS}/\text{PIC71VR}) \\ & + \text{JW}((\text{DUMTAXREFORM} * (\text{APPLMTGNWRPRIVNS} \\ & + \text{APPLMTGNWRCMHCNS}/\text{PICR71VR})) \end{aligned}$$

The sum of the coefficients on mortgage approvals for single and multiple starts indicates that since tax reform an additional \$1 million in approvals has financed 19.5 single starts and 46 multiple starts.

Mortgage approvals by private lending institutions is a function of total mortgage approvals by the individual category of lending institution. The proportion of lending allocated to residential mortgages responds negatively to an aggregate vacancy rate and to the level of non-residential construction and CMHC approvals. The mortgage approvals of individual categories of financial institutions are influenced by such variables as permanent income and mortgage and other interest rates. Some of these influences work indirectly through their impact on the total assets of the lending institutions. Institutional aspects of mortgage lending are also taken into account.

Completions and expenditures are modelled as distributed lags on starts. The stock of existing houses is included in the residential construction expenditure equation to capture the effects of additions and alterations to existing residences plus commissions on the sale of existing homes.

### 3.6.7 MTFM

The MTFM housing sector treats single detached and multiple unit housing separately. Single unit housing is assumed to be owner occupied and multiple unit largely rental.

The key price equation is for Multiple Listing Service housing (PHMLS). It is determined by equating stock demand and supply for housing. Its form is:

$$\begin{aligned} & \text{PHMLS/PCPI} * (.05 + \text{RHPTR} + \text{RMCM}/100 * \text{IHLVES} + \text{RL10IND}/100 \\ & * (1 - \text{IHLVES})) + \text{PHIUR/PCPI} \\ & = \text{A0} + \text{A1} * \text{YPERM/HP15} + \text{A2} * \text{KHS} (-1)/\text{HP15} \\ & + (\text{JW}(\text{J1D}(\text{PHMLS}(-1))^4)/\text{PCPI} \end{aligned}$$

where PCPI is the CPI, .05 is the depreciation rate, RMCM is the conventional mortgage rate, IHLVES is the loan to value ratio, RL10IND is the McLeod, Young, Weir 10 industrial bond rate, PHIUR is the cost of insurance, utilities and repairs, YPERM is permanent income, HP15 is the population over age 15, KHS is the end-of-period stock of single detached housing, and the last term reflecting the expected price appreciation of a house.

The price of new housing, which is important in determining housing starts, is specified to be a distributed lag on the price of Multiple Listing Service housing.

The equations for housing starts depend primarily on the factors that affect the expected profitability of constructing housing. The key equation for single starts (IHSS) is:

$$\text{LOG}(\text{IHSS}) = \text{H0} + \text{H1} * \text{LOG}(\text{PHNHPI}) + \text{H2} * \text{LOG}(\text{PISCR}) + \text{H3}$$

$$* RPRIME + H4 * J4P(PHNHPI) + H5 * J1D(UR)$$

where PHNHPI is the price of new housing, PISCR is an input price index for residential construction, RPRIME is the chartered bank prime lending rate, and UR is the unemployment rate.

The equation for multiple unit starts (IHMS) is:

$$\begin{aligned} & (1) \\ IHMS - IHMSOC = & J0 + J1 * DHPMRT * PHRENTS/UCRM \\ & (2) \\ & + J2 * (1 - DHPMRT) * PHNHPI/PISCR \\ & (3) \\ & + J3 * (RMCM - J12A(RMCM)/J12A(RMCM)) \end{aligned}$$

where IHMSOC is the number of government supported housing starts, DHPMRT is rental units as a proportion of multiple units, PHRENTS is the rental price index, UCRM is a user cost of rental variable including depreciation, property taxes, interest payments and reflecting corporate tax regulations, and RMCM is the conventional mortgage rate.

Term (1) reflects the profitability of constructing multiple units for rental and term (2) as owner occupied units.

Expenditures on new housing are determined by a weighted average of a distributed lag of single and multiple unit housing starts. Other expenditures are modelled in per capita terms as a function of income, the mortgage rate, the real price of housing and the unemployment rate.

### 3.6.8 QFS

The QFS housing sector was borrowed directly from RDXF with two important modifications in the MLS price and the "other" component of residential construction. The QFS housing sector has an equation for total housing starts and the split between singles and multiples is specified exogenously. The important price equations are for Multiple Listing Service housing and construction cost per unit for single detached dwellings.

The equation for the Multiple Listing Service price is the following:

$$\begin{aligned}
 \text{LOG(PMLS/PCPI)} = & -3.39069 - .01160*Q1 - .02218*Q2 - .03745*Q3 \\
 & \{ \\
 & - (1 - .51659) * .158 * \text{LOG(PCPIHTAX/PCPI)} \\
 & - (1 - .51659) * .039 * \text{LOG(PCPIHI/PCPI)} \\
 & \quad (1) \\
 & - (1 - .51659) * .278 * (\text{RMC} * .66 + \text{RMYWI} \\
 & * (1 - .66) - \text{PCPIP4})/100 \\
 & - (1 - .51659) * .112 * \text{LOG(PCPIHM/PCPI)} \\
 & \quad \} \\
 & - (1 - .51659) * .224 * \text{LOG(PCPIEN/PCPI)} \\
 & \quad (2) \\
 & + 1.11549 * \text{LOG(YPD71PK)} \\
 & \quad (3) \\
 & - 2.65264 * \text{LOG(KHST(-1)/LFPOP)} \\
 & \quad (4) \\
 & + .10470 * \text{J4P(PMLS(-1)/PCPI(-1))} \\
 & \quad (5) \\
 & + .51659 * \text{LOG(PMLS(-1)/PCPI(-1))}
 \end{aligned}$$

where PMLS is the price of Multiple Listing Services houses,  
 PCPI is the consumer price index, Q1, Q2, and Q3 are  
 quarterly dummy variables, PCPIHTAX is the CPI property  
 tax component, PCPIHI is the CPI component for homeowners  
 insurance, RMC is the conventional mortgage rate,  
 RMYWI is the McLeod, Young, Weir industrial bond average,  
 PCPIP4 is the year-over-year percentage change in the CPI,

PCPIHM is the CPI component for homeowners repairs and maintenance,

PCPIEN is the CPI component for energy,  
 YPD71PK is real permanent disposable income per capita,  
 KHST is the housing stock in units, and LFPOP is the labour  
 force population.

The equation is specified in terms of the Multiple Listing Service Price Relative to the CPI. The first set of terms in the equation following the constant term and the seasonal dummies represent the user cost of housing including property tax, homeowners insurance, real mortgage interest, repairs and maintenance, and energy. The sum of weights in the components of user cost sum to less than 1 because the share of depreciation costs is excluded. The user cost exerts a depressing influence on the relative price of MLS housing. This effect is not estimated but is imposed a priori. The second term is for real permanent disposable income per capita. It reflects demand considerations and has a positive coefficient. The third term measures the stock of housing per member of the labour force. It is a supply variable which serves to decrease the relative price of housing. The fourth term is the percentage change in the relative price of housing. It can be viewed as an expectational term raising demand and tending to increase the relative price of housing. The fifth term is the lagged LHS variable. It allows for a lagged response to the RHS variables except for the user cost components.

The "other" component of residential investment, which is composed mainly of renovations and real estate commissions, is a function of real permanent income per capita, the nominal commercial paper rate, and the ratio of MLS price to the CPI.

### 3.6.9 RDXF

The key equations in the RDXF housing sector are for total housing starts and the Multiple Listing Service price per unit of housing. The housing starts (HST) equation is a supply function and is as follows:

$$\begin{aligned} \text{LOG(HST)} = & 4.64363 - .59473 * Q1 + .07402 * Q2 + .10942 * Q3 \\ & + 1.28078 * \text{LOG(PMLS)} - .94133 * \text{LOG(PHCC)} \\ & + .22469 * \text{DUMMY1} - .07980 * \text{J1L(RPRIME)} \\ & + .16895 * \text{DUMMY2} \end{aligned}$$

where Q1, Q2, and Q3 are quarterly dummy variables, PMLS is the multiple listing service price, PHCC is per unit construction costs, DUMMY1 is a dummy variable equal to 1 in 78Q1 and to -1 in 78Q2 to capture MURBs, DUMMY2 is a dummy variable equal to 1 in 67Q2, 70Q3, 70Q4, and 75Q4, capturing increases in publicly financed starts

in these quarters, and RPRIME is the chartered bank prime lending rate reflecting financing costs.

The multiple listing price equation (PMLS) is based on the assumption that the MLS housing price adjusts to reduce the excess demand for housing. It is thus specified as responding to the gap between stock housing demand and supply per capita. Demand is in terms of the desired stock which is said to be a function of real effective purchasing power per capita, the Multiple Listing Service Price and the conventional mortgage rate. The exact form of the equation is:

$$\begin{aligned} \text{LOG(PMLS/PCPI)} = & 6.40588 * Q1 + 6.38490 * Q2 + 6.37656 * Q3 \\ & + 6.41372 * Q4 + .94583 * \text{LOG(UDPADJ/NPOP)} \\ & + .25947 * \text{LOG(PHCC/PCPI)} + .01330 + \text{J1L(RPRIME)} \\ & + .54239 * \text{J1L(LOG(PMLS/PCPI))} \\ & - .00968 * \text{J1L(RMC)} - 2.35415 * \text{LOG(J1L(SHT)/NPOP)} \end{aligned}$$

where PCPI is the consumer price index, Q1, Q2, Q3, and Q4 are quarterly dummy variables, UDPADJ is real effective purchasing power, RMC is the conventional mortgage rate, NPOP is population, and SHT is the stock of housing units.

The split between singles and multiples is exogenous and is based on historical shares. RDXF incorporates equations for completions and expenditures based on the traditional lagged relationships. It also has an equation for alterations and improvements as a function of real effective purchasing power, energy costs, the mortgage rate, and the price of Multiple Listing Service Housing relative to the price of home repairs.

### **3.6.10 MACE**

Housing investment is not disaggregated. It is modelled as part of business fixed investment.

### **3.6.11 SAM**

Housing investment is also treated as part of business fixed investment. The housing stock is counted as part of the data for producing capital, and housing services are produced (like any other good) as part of the non-energy good. Households thus own their housing, as they do all other real capital, through equity claims.



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### 3.7 Consumption and the Savings Rate

#### **3.7.1 CANDIDE 2.0**

Aggregate consumption is determined residually in CANDIDE. Two savings equations - one for discretionary saving and one for private contractual saving - are determined stochastically. The detailed composition of consumption, which is required for the production side of the model, is modelled in some 48 equations. Each individual consumption category measured in per capita or per household terms is explained by: 1) total consumption similarly measured and net of imputed items, 2) the price of the category relative to the overall consumer price, and 3) other variables peculiar to each equation. The equations allow for demographic factors, stock effects for consumer durable and for the impact of habit formation on expenditures.

The personal contractual saving (SPC\$) equation is:

$$\begin{aligned} & \text{(1)} \\ \text{SPC\$}/\text{PFCPIP} = & -1474.94 - .44506 * ((\text{GRPENQ.DT\$} + \text{GRPENC.DT\$} \\ & + \text{GRF.DT.PCPEN\$} + \text{GRP.DT.PCPEN\$} \\ & + \text{GRP.DT.PCVPS})/\text{PFCPIP}) \\ & \text{(2)} \\ & + .16892 * (\text{C.RRSPMAX}(-1)/\text{PFCPIP}(-1)) \\ & \text{(3)} \\ & + .074260 * ((\text{PDY\$} - \text{.INVFARM\$} - \text{CSE40\$} \\ & - \text{CSM20\$})/\text{PFCPIP}) \end{aligned}$$

where PFCPIP is the deflator for the permanent income proxy, term (1) represents contributions to the various government pension plans, term (2) captures RRSP savings incentives, and term (3) is real income as measured by personal disposable income minus farm inventories and health and education expenditures and divided by the permanent income proxy deflator.

The negative sign on the coefficient for contributions to government pension plans reflects the fact that such contributions are not counted as private savings, but are actually in many respects comparable. To the extent that the coefficient is less in absolute value than minus 1, however, government plan saving and personal contractual saving are not perfect substitutes. It is surprising that term (2), the RRSP term, has a positive effect on contractual saving, since it is not contractual.

The personal discretionary saving (SDISS) equation is:

$$\begin{aligned}
 & \text{(1)} \\
 \text{SDISS/PFCPIP} = & -4449.42 + .0050675 * (\text{J1D}(\text{DURATE}) \\
 & * ((\text{PDY\$} - .\text{INVFARM\$} - \text{CSE40\$} - \text{CSM20\$})/\text{PFCPIP}) \\
 & \text{(2)} \\
 & + 134.79 * \text{J1P}(\text{PFCPIP}) \\
 & \text{(3)} \\
 & + \text{JW}(\text{C.RRSPMAX}/\text{PFCPIP}) \\
 & \text{(4)} \\
 & + \text{JW}((\text{PDY\$} - .\text{INVFARM} - \text{CSE40\$} - \text{CSM20\$})/\text{PFCPIP})
 \end{aligned}$$

where DURATE is the unemployment rate, and the other variables are as defined above.

Term (1) indicates that increases in the unemployment rate interact with disposable income to increase saving. Term (2) shows that inflation increases personal discretionary saving. Term (3) captures the impact of RRSP on discretionary saving. The sum of the lag weights is 1.5186 indicating that discretionary saving goes up by more than one-for-one with the RRSP maximum. The sum of the lag weights on real personal disposable income, term (4) is .02380.

Neither savings equation includes an interest rate variable.

### 3.7.2 TIM

The consumption sector of TIM is disaggregated into over 80 categories. Its specification is characterized as Houthakker-Taylor in that per capita real consumption of goods or services is specified as a function of its past value (serving as a proxy for habit persistence), per capita real total expenditures or disposable income and the price of the good relative to other consumer goods prices or the CPI.

An important concept in TIM is discretionary income. It is defined as disposable income minus expenditures on necessities such as food, clothing, and shelter. This is the income variable

that is the most important determinant of spending on other items such as cars, furniture, recreation and travel.

The typical specification for an equation for a consumption necessity is:

$$C_{Ni}/POP = a_0 + a_1 + C_{Ni}(-1)/POP(-1) + a_2 * YD/POP + a_3 * Z$$

where  $C_{Ni}$  is consumption on category  $i$ ,  $POP$  is population,  $YD$  is disposable income, and  $Z$  is other variables such as demographics or multiple or single housing stock variables in energy or rental consumption categories

The typical specification for other consumption categories including durables is:

$$C_{Oi}/POP = a_0 + a_1 * C_{Oi}(-1)/POP(-1) + a_2 * YDISK/POP \\ + a_3 * CO_{Pi}/YDISP + a_4 * Z$$

where  $CO_i$  is consumption on category  $i$  which is other than a necessity,  $POP$  is either population or number of households,  $YDISK$  is discretionary income,  $CO_{Pi}$  is the price deflator for category  $i$ ,  $YDISP$  is the price deflator for discretionary income, and  $Z$  is other variables.

### 3.7.3 RDX2

The consumption categories modelled in RDX2 were 1) motor vehicles and parts; 2) other durables; 3) non-durables and semi-durables; 4) services excluding rent; and 5) gross rent. The general form of the consumption function estimated was:

$$C/NPOPT = a_0 + a_1 * YPERM + a_2 * VR + a_3 * PEX + a_4 * CR \\ + a_5 * TR + a_6 * RELPR * YPERM + a_7 * R$$

where  $C$  is consumer expenditures,  $NPOPT$  is population,  $YPERM$  is permanent income,  $VR$  is real wealth per capita,  $PEX$  is price expectations,  $CR$  reflects credit availability,  $TR$  is real disposable income,  $RELPR$  is a vector of relative prices, and  $R$  is a real interest rate variable.

For durables the specification estimated was similar except that it was the stock that was related to the explanatory variables and the equation was in stock adjustment form.

The equations actually included in the model embodied much simpler specifications since many of the variables tried did not pass statistical tests.

The equation for consumption of motor vehicles specified per capita consumer expenditures as a function of permanent income, a relative price term constructed by multiplying the relative price of motor vehicles times permanent income, and the per capita lagged stock.

The equation for other durables includes non-wage permanent income, real wealth per capita, investment in residential construction per capita, the relative price term and the per capita lagged stock.

The equation for non-durables and semi-durables is simpler. It specifies the fourth difference of consumption per capita to be a function of the fourth difference of permanent income and government transfers per capita. The marginal propensity to consume is higher out of transfers.

The equation for services excluding rent is also in fourth difference form. The explanatory variables are permanent income, real provincial medicare payments per capita, and dummy variables for the transfer of hospitals to the public sector and Expo 67. The equation also has a lagged dependent variable.

Consumer expenditure on rent is tied to the housing stock.

Interest rates had no role in the consumption sector of the latest version of RDX2. This contrasted with the original version of RDX2 where interest rates could be found in the equations for durables, and non-durables and semi-durables.

### **3.7.4 CHASE**

CHASE has consumer demand equations for motor vehicles, household durables, other durables, semi-durables, food and beverages, energy, other non-durables, services and rent. The general specification has expenditure as a logarithmic function of real discretionary income, population and relative prices. The concept of discretionary income utilized is taken from RDXF and is defined as disposable income minus personal transfers to corporations and the inflation premium on net personal wealth. Particular equations depart from the general specification.

Consumer expenditures on motor vehicles is broken down into new passenger cars and other. New passenger car sales in thousands of vehicles and divided by employment is specified as a function of the price of new cars, real effective purchasing power per member of the employed labour force, the unemployment rate, the prime rate and dummy variables for car rebates, sales tax rebates and auto strikes. Real consumer expenditure on new cars is tied to the number of new

car sales by a percentage change relationship. Consumer expenditures on motor vehicles and parts other than new cars per employed member of the labour force is explained by real effective purchasing power per employee, the stock of motor vehicles and parts per employee, the price of motor vehicles and parts relative to the price of new automobiles, the real interest rate defined as prime rate minus the expected change in the CPI, and dummy variables for rebates and strikes.

The equation for consumer expenditures on household furniture and appliances follows the general specification with the addition of housing starts and the real interest rate and distributed lags on all of the variables.

The equation for other durables is estimated in percentage change form with the one quarter lagged value of real effective purchasing power per capita and the lagged stock per capita also included.

The semi-durable equation includes the lagged left hand side variable as do the equations for non-durables excluding food and energy. The consumer energy demand equation also has dummy variables reflecting the temperature and oil supply disruptions.

The consumer expenditure on food and beverages equation has two quarter lags on relative prices and real effective purchasing power per capita.

The consumer expenditure on service equation has a six quarter lag on real effective purchasing power per capita and also includes the lag of the consumer price index divided by its value four quarters earlier.

The rent equation is a function of lagged housing stock per capita.

### **3.7.5 DRI**

The DRI model breaks out eleven consumption categories and has a separate equation for the number of cars sold. The equations generally follow the Friedman permanent income model with consumption specified as a function of permanent and transitory income per person 15 years and over and relative prices. Since the purchase of durable and semi-durable goods is a form of savings, these categories benefit from a larger than proportional share of spending out of transitory income.

The equation for a number of new cars sold specifies car sales per person over 15 as a function of the stock of cars per person over 15, permanent and transitory income, a number of dummy variables to reflect rebates and discounts, and an index of the user cost of cars. The latter variable is calculated using the price of autos, the 90 day finance paper rate, a quarterly depreciation rate of 6 percent, and the price of gasoline relative to other goods and services. Expenditures on

motor vehicles and parts are expressed as car sales times unit value which in turn is a function of permanent and transitory income, and the relative price of cars.

Real expenditure on furniture and appliances has no relative price term. Expenditure on other durables follows the standard specification. Expenditure on clothing and footwear has no relative price term. Expenditures on other semi-durables, food, fuel, other non-durables, medical care, and other services do not include a transitory income term. Expenditure on rent and imputed rent is a function of the constant dollar housing stock and the mortgage rate. The stock of cars and housing is included as an additional explanatory variable in the consumer expenditures on fuel equation.

### **3.7.6 FOCUS**

The FOCUS model also follows a permanent income approach to modelling consumption. Equations are estimated for new passenger car sales in units, real consumer expenditures on new and used automobiles, other durable goods, semi-durables, non-durables and services.

The equation for new passenger car sales is relatively sophisticated. Sales are a function of real permanent income per household, population age 16 and over and a distributed lag on changes in transitory income. Other important explanatory variables include the increase in the unemployment rate adjusted for changes in the UI regime, the user cost of owning and operating a car relative to the CPI, and the rate of change in the CPI one quarter in the future reflecting advance purchase in anticipation of price increases. The lagged stock of cars is also included in the equation reflecting the stock adjustment nature of the demand for cars. The user cost variable is made up of 5 components; 1) the price of cars; 2) depreciation in value and demolition; 3) financing costs as measured by the 90 day finance company paper rate; 4) capital gains and losses on cars; and 5) gasoline expenses.

Real personal consumer expenditures on cars is a function of car sales and a time trend. The equation for other durables depends on the number of households, permanent and transitory income, the price of other durables relative to the overall deflator for consumer spending, and the number of new housing completions, plus conversions net of demolitions.

Real consumer expenditures on semi-durable goods is a function of the number of households, permanent and transitory income, and the relative price of semi-durable goods.

Real consumer expenditures on non-durable goods is a function of the same variables with the addition of the number of cars, the stock of houses, and the price of gasoline.

Real consumer spending on services is adjusted to include hospital insurance expenditures. Explanatory variables are the number of houses, permanent and transitory income, and the stock of cars (to capture auto-related services). There is no special treatment of rent.

### 3.7.7 MTFM

There are eight categories of consumer expenditures in MTFM: 1) automobiles and parts; 2) other durables; 3) semi-durables; 4) food, beverages, and tobacco; 5) energy; 6) other non-durables;

7) gross paid and imputed rent; and 8) other services. The consumer expenditure equations, which are based on the approach of Juster and Wachtel {F.T. Juster and P. Wachtel, [Op. Cit.]}, portray per capita consumption as a function of permanent income, transitory income, and relative prices. In addition, a number of other variables such as the real rate of interest, demographic variables, and consumer confidence as proxied by inflation and the unemployment rate are included in individual equations. Many of the equations are specified in logarithmic form.

The equation for expenditures on automobiles, repairs and parts per capita has as explanatory variables per capita permanent and transitory income, the price of cars relative to the consumer expenditure deflator, the real rate of interest, the unemployment rate, the lagged rate of inflation, and the per capita stock of automobiles.

The other durables equation is similarly eclectic and includes real permanent and transitory income per capita, the price of durables relative to the consumer expenditure deflator, the real interest rate, the change in the unemployment rate, and the per capita stock of other durables.

The equation for semi-durables is in logarithmic form and in addition to the traditional variables includes the real interest rate, and the change in the unemployment rate.

The equations for non-durables are in logarithmic form and follow the traditional specification except for the equation for the consumer expenditure on food, beverages, and tobacco which includes the real interest rate. The equation for services other than rent is the same except that it includes the inflation rate as well as the real interest rate.

The equation for rent is a technical relationship tied to the housing stock.

### 3.7.8 QFS

The consumption equations in QFS generally specify consumption per capita to be a function of permanent income and relative prices. The majority of the consumption equations are corrected for autocorrelation.

The equation for durables excluding new autos adds to the basic specification per capita housing starts and the stock of other durables.

The equation for consumer expenditures on new automobiles also incorporates the user cost of cars, the per capita stock of autos, transitory income, the unemployment rate, and dummy

variables for manufacturer and sales tax rebates.

The equation for semi-durables includes transitory income and a dummy variable for the sales tax rebate.

The equation for expenditures on food and beverages has a transitory income term as does the equation for other non-durables.

The equations for consumer expenditure on fuel and services follow the standard format.

The equation for consumer expenditure on imputed and paid rent is a function of the lagged housing stock.

The interest rate is not included directly as an explanatory variable in any of the consumption equations. It enters indirectly through the user cost for cars which is included in the equation for new automobiles. This variable is defined as the price of cars times the McLeod, Young, Weir yield on industrial bonds minus the expected increase of new car prices over the next four quarters, plus the depreciation on existing cars.

The long-run marginal propensity to consume is reportedly .6 to .7. This is significantly lower than in most of the other models.

### **3.7.9 RDXF**

In RDXF consumption is disaggregated into nine components: 1) household durables; 2) motor vehicles; 3) miscellaneous durables; 4) semi-durables; 5) food; 6) energy; 7) other non-durables; 8) services excluding rent; and 9) rent. The general specification utilized is:

$$\text{LOG}(C_i/N) = a_0 + a_1 * \text{LOG}(U/N) + a_2 * \text{LOG}(P_i/P)$$

where  $C_i$  is consumption expenditures in category  $i$ ,  $N$  is population,

$U$  is real effective purchasing power,  $P_i$  is the price applicable to category  $i$ , and  $P$  is the consumer price index.

Most of the equations also include the lagged dependent variable. They could thus be characterized as conforming to the permanent income or habit persistence hypothesis. The overall income elasticity of consumption is close to unity. The income term, real effective purchasing power, is defined as real personal disposable income excluding the inflation premium on interest

payments and current transfers to corporations. The derivation of the inflation premium was discussed above in the section on wealth and permanent income.

The equation for household durables also includes the lagged per capita stock of household



durables and investment in residential construction per capita.

Consumer expenditure on motor vehicles is modelled within a stock adjustment framework. The stock of motor vehicles held by consumers is specified to be a function of the desired stock per capita and the lagged actual depreciated stock. The desired stock per capita is determined as an exponential function of real effective purchasing power per capita and user cost of motor vehicles. Consumer expenditures on motor vehicles are the difference between the current period stock and the depreciated lagged stock.

The equation for other durables includes the lagged per capita stock of other durables and the ratio of labour force population to total population. The equation for semi-durables also includes the ratio of labour force to population.

The equation for energy consumption follows the standard specification with the addition of two dummy variables to capture shifts in demand. The equation for expenditures on food and beverages includes the ratio of the total labour force to total population as an additional explanatory variable. The equation for other non-durables has a five quarter lag on per capita real effective purchasing power, and a dummy variable for the second quarter of 1971.

Per capita consumer expenditure on rent is tied to the per capita housing supply. The equation for consumer expenditures on services only contains real effective purchasing power.

The only place that interest rates enter directly into the RDXF consumption sector is in the user cost of capital term which is important in explaining demand for motor vehicles. The interest rate utilized is real and is defined as the prime rate minus expected consumer price inflation.

### **3.7.10 MACE**

Consumer spending in MACE is modelled by a single equation. The theory underlying this equation is the life cycle model. Consumption is a function of after-tax wage and transfer income and the real value of household wealth. The specific equation is:

$$C/Np = .82233 * (Ne * We + Nne * Wwe + Gt) * (1 - rtp)/(Pc * Np) \\ + .021237 * J2A(V/pcNp) + .42639$$

where C is personal expenditure, Np is population, Ne and Nne is energy and non-energy employment, We and Wne are the wage rates in energy and non-energy sectors, Gt is government transfers, rtp is the total personal income tax rate, Pc is the implicit consumption price deflator, and V is the market value of private sector wealth.

The marginal propensity to spend disposable wage and transfer income is .82. It is estimated in the MACE documentation that a 1 percentage point increase in short-term interest rates will lower the market value of wealth by 5 per cent. This would lower consumption by one quarter per cent.

### 3.7.11 SAM

Consumption and labour supply in SAM are assumed to result from the same intertemporal utility maximization decision over an infinite time horizon. Utility is specified as a log-linear function of consumption (C) and leisure (L). It is posited that the economy is made up of identical households consisting of a head and  $N_k$  dependents. The result of such an optimization is a specification in which consumption is proportional to total real wealth, human and financial. Taxation is also taken into account. The equation actually estimated is:

$$\begin{aligned} \text{CON/NPOP} = & \{a_1 * ((VT/PC + VHPVN)/NPOP)/(1 + a_2 * (1 - RATA\text{XN}) \\ & * ((1 + NK)^{a_3})/(1 - RMT\text{AX}))\} + \{a_5 * (\text{LOG}(PS) \\ & - \text{LOG}(PC)) * ((VT/PC + VN\text{SS})/NPOP)\} \\ & + (a_6 * (VHPV - VHPVN))/NPOP + a_4 * \text{DUMMY} \end{aligned}$$

where CON is consumer expenditures, NPOP is the adult population, VT is the end of period financial wealth, PC is the consumption price index, VHPVN is human wealth valued at steady state real wage and current employment, RATA $\text{XN}$  is the average personal income tax rate, RMT $\text{AX}$  is the marginal tax rate on personal income, NK is the dependency ratio, PS is the steady state consumption price index, VHPV is the present value of human wealth, and DUMMY is a dummy variable equal to 1 after 1969.

Term (1) captures the proportionality between consumption and total wealth taking into account taxes. Term (2) represents the extent of excess real money balances, that is the difference between actual and desired real balances. Households use money stocks to buffer shocks and willingly move off their desired position in the short-run when responding to shocks. However, to the extent that excess balances are held, households will raise consumption temporarily. This temporary asset effect is accompanied by a similar constraint for human wealth (Term 3) which can be regarded as similar to a transitory income variable.

## 3.8 Labour Supply

### 3.8.1 CANDIDE 2.0

The CANDIDE 2.0 model has a detailed demographic block. It includes: 1) fertility rate equations; 2) population cohort equations by age and sex; 3) participation rate equations by age and sex; 4) school enrollment equations; and 5) family formation equations.

The most important equations are for participation rates by aggregated male and female cohorts. They have the following general form:

(1)

$$\begin{aligned} \text{DiPARTRATE}_{j,k} = & a_0 + a_1 * (W/\text{CPI}) * (1 - (\text{GR.DT.P\$}/\text{PY\$})) \\ & + a_2 * \text{DURATE} + a_3 * \text{DUIDUMMY} \\ & + a_4 * \text{"PENSION"} + a_5 * \text{"DEMOG"} \end{aligned}$$

where  $i$  is M or F for male or female,  $j,k$  is the age group i.e., 14.19, 20.24, 25.54, and 55+ for males and 14.24, 25.44, and 45+ for females,  $W$  is the aggregate wage rate,  $\text{CPI}$  is the consumer price index,  $\text{GR.DT.P\$}$  is total personal taxes,  $\text{PY\$}$  is personal income,  $\text{DURATE}$  is the unemployment rate,  $\text{DUIDUMMY}$  is the dummy variable for the 1971 modifications to the UI regime, "PENSION" is a variable for real pension benefits per capita, and "DEMOG" represents demographic variables particular to age and sex groups.

The part rate is influenced by changes in the after-tax real wage as represented by term (1). This term stands for the neoclassical work/leisure choice. It has a positive sign in all three of the female participation rate equations and on all of the male categories in the long-run, except the age 25 to 54 cohort. An increase in the after-tax real wage causes individuals to enter the labour force. Similarly, an increase in taxes causes individuals to leave the labour force. This offsets the impact of tax changes on employment.

The unemployment rate has a negative sign in the equation for females age 14 to 24 and over 45 and for males 14 to 19 and 20 to 24 suggesting a discouraged worker effect. It has a positive sign in the equation for females 25 to 44 and males over 55 as suggested by the additional worker hypothesis. The UI dummy variable captures increased participation by some age and sex

groups subsequent to the 1971 revisions to unemployment insurance. The "PENSION" variable for real pension benefits has a significant effect in reducing participation of males over 55. The demographic variables included for particular age-sex groups include child-to-female ratio for women of child-bearing age and school enrollment rates for young people.

### 3.8.2 TIM

TIM also has a detailed demographic block. Total population under 70 is disaggregated into five-year age cohorts for both sexes and a separate group for infants under 1 year old. The demographic block can be solved separately from the rest of the model since it does not rely on economic variables. The age and sex groups generated by the demographic block provide the source population for labour supply.

TIM contains participation rates for twelve age/sex categories. The part rate for prime age males is exogenous. Total unemployment insurance plus wages in real terms has a positive impact on participation in most categories. This is similar to CANDIDE except that it recognizes that unemployment insurance benefits are an inducement to labour force participation and is before-tax rather than after-tax. The impact of unemployment varies from one category to another, but for most age groups it discourages them from entering the labour force. An exception is older women where the additional worker effect predominates. The participation of workers of retirement age is affected by the size of pension benefits.

### 3.8.3 RDX2

Both the original and latest versions of RDX2 only had one equation to explain the participation rate. The form of the equation in the latest version of RDX2 is:

$$\begin{aligned}
 100 * J1D(NL/NPOP) = & .17439 - .72087 * QC1 + .54011 * QC2 \\
 & (1) \\
 & + 1.0174 * QC3 - .54326 * (Q1 + Q2) * (100 * J1D(NPOPSS/NPOP)) \\
 & (2) \\
 & - .80641 * (Q3 + Q4) * (100 * J1D(NPOPSS/NPOP)) \\
 & (3) \\
 & + 4.4537 * J1D(UGPPA/UGPPD) \\
 & (4) \\
 & + 34.819 * J1D(J19S(NIMS - NEMS)/NPOP)
 \end{aligned}$$

where NL is labour force, NPOP is population, QC1, QC2, and QC3 are constrained quarterly dummies, NPOPSS is the population 14 and over attending school, UGPPA/UGPPD is RDX2's measure of capacity utilization,

the ratio of actual output, adjusted for unintended inventory changes, to desired, NIMS is the number of immigrants, and NEMS is the number of emigrants.

Terms (1) and (2) capture the impact of school attendance on participation rates. Term (3) is the cyclical variable. The magnitude of its coefficient indicates a relatively small discouraged worker effect. Term (4) allows for the greater than average tendency of immigrants to enter the labour force.

The participation rate equation in the original version of RDX2 was similar. The only difference was the inclusion of the change in per capita real disposable income as the cyclical variable.

In RDX2 it is possible to increase the supply of labour by increasing net immigration. The equations for the number of immigrants and the number of emigrants are thus important.

### **3.8.4 CHASE**

CHASE has a population sector based on Statistics Canada population projections. This provides the source population for the labour force calculations. The groups utilized are age 15-24, 25-54, and 55 and over for both males and females. The participation rate equations are very simple and are of the form:

$$\text{LOG}(\text{RNLIjCA}) = c_0 + c_1 * \text{QTIMECA} + c_2 * \text{JIP}(\text{NECA})$$

where RNLIjCA is the participation rate for sex i and age group j,  
QTIMECA is a time trend, and NECA is total employment.

A problem with this specification is that it relates the level of the log of the part-rate (instead of the more appropriate change in the log) to the percentage change in employment. This implies that there is no ongoing discouraged or additional worker effects. The actual estimated coefficients on the percentage change in employment are very small. Thus in practice even very large percentage changes in employment have only minimal impact on part-rates.

### **3.8.5 DRI**

Labour supply is not modelled in detail in the DRI model. The model has equations for employment and the unemployment rate. Labour force is calculated by dividing employment by one minus the unemployment rate. Alternatively, the participation rate can be set exogenously and the unemployment rate determined residually.

The unemployment rate equation used to derive the labour force is an Okun's law specification in which the gap between the actual and full employment unemployment rate is a function of

the gap between actual and potential GNP and of the ratio of prime age males to the total working population. The precise equation for the unemployment rate (RU) is:

$$RU = RUF E + 66.0630 - 23.4119 * UCAP - 17.2082 * UCAP(-1) \\ - 11.2383 * UCAP(-2) - 5.50222 * UCAP(-3) - 35.0650 \\ * (NM25@@54/(N15\& - N65\&))$$

where RUF E is the full employment unemployment rate, UCAP is the gap between actual and potential GNE defined as GNP71/GNP71FE, NM25@@54 is the prime age male labour force, and N15\& - N65\& is the total working population.

The sum of the coefficients on the lagged output gap is 59.77. This means that a 1 per cent increase in output would lower the unemployment rate by .6 percent.

### 3.8.6 FOCUS

FOCUS has labour force source population for men and women broken down into three age groups - 15 to 24, 25 to 54, and over 55. Labour force is derived using participation rate equations. The general form of the equation is:

$$RP_{ijVR} = a_0 + a_1 * E/N15\& + a_2 * DMYTIME \\ + a_3 * MAXWBPAYUI/AAWPS/40 * a_4 * OAS/CPINSVR$$

where  $RP_{ijVR}$  is the participation rate for sex  $i$  and age group  $j$ ,  $E/N15\&$  is the employment population ratio,  $DMYTIME$  is a time trend,  $MAXWBPAYUI/AAWPS/40$  is maximum unemployment insurance benefits as a proportion of the average wage, and  $OAS/CPINSVR$  is real old age security payments.

The most important explanatory variable is the employment population ratio. Its coefficient indicates a discouraged worker effect for all age/sex groups. This effect ranges from .05 for prime age males to .82 for young females. The average impact (using 1976 average population weights) is .4. Thus a one percent increase in employment would raise the labour force by .4 per cent.

The time trend is positive for female participation rates except for the oldest group and negative for males except for the youngest group. The unemployment insurance variable only enters

in the equation for prime age women. The Old Age Security benefits variable is only included in the equations for the oldest age group.

### 3.8.7 MTFM

MTFM has the same breakdown of labour force source population as FOCUS. The participation rate equations for each group include; 1) the employment-population ratio for the group; 2) the employment-population ratio for all of the other groups; and 3) other variables. The general form of the equation is:

$$\begin{aligned} \text{LOG(LPR}_{ij}) = & a_0 + a_1 * \text{LOG}(E_{ij}/LP_{ij}) + a_2 * ((E - E_{ij})/(LP - LP_{ij})) \\ & + a_3 * \text{OTHER} \end{aligned}$$

where  $LPR_{ij}$  is the participation rate for sex  $i$  and age group  $j$ ,  
 $E_{ij}$  is employment for sex  $i$  and age group  $j$ , and  
 OTHER represents other variables.

The other variables include school enrollment rates for younger people, positive time trends for all but prime age males, and the ratio of transitory to permanent income for prime age males, which discourages participation.

The discouraged worker effects captured by the coefficient on the own employment-to-population rate are particularly strong. The elasticity coefficient for prime age males at .4 is the lowest, other coefficients approach or even exceed 1.

### 3.8.8 QFS

Labour force source population is disaggregated into four groups in QFS - males and females under 25 and 25 and over. The participation rate equations reflect the discouraged worker hypothesis and take the following form:

$$\text{LFPR}_{ij} = a_0 + a_1 * \text{LFEDEV}(-1) + a_2 * \text{QUIC} + a_3 * \text{QTIME}$$

where  $\text{LFPR}_{ij}$  is the participation rate for sex  $i$  and age group  $j$ ,  
 $\text{LFEDEV}$  is the percentage deviation of employment from an exogenous trend, and  $\text{QUIC}$  is a dummy variable equal to 1 from 72Q1 onwards reflecting the revision to the Unemployment Insurance regime.

The coefficient on the deviation of employment from trend is quite high for young people (.81 for men and .71 for women) but relatively low for prime age adults (.15 for men and .24 for women).

The dummy variable for the unemployment insurance revisions lowers participation rates for prime age men and women and raises participation rates for young people. The time trend is positive except for prime age males.

### 3.8.9 RDXF

RDXF only has a single participation rate equation, but it is quite complex. It can be summarized as follows:

$$\begin{aligned} \text{LOG(NL/NPOP)} = & \text{F(QTIME)} + .20088 * \text{J2A}(\text{LOG(NE/NPOP)}) \\ & - \text{F(QTIME)} - (\text{RNUTO}/100) - .0035) \\ & - .18075 * \text{J2L}(\text{J2A}(\text{LOG(NE/NPOP)} - \text{F(QTIME)}) \\ & - (\text{RNUTO}/100) - .0035)) \\ & + .10823 * \text{J4L}(\text{J2A}(\text{LOG(NE/NPOP)} - \text{F(QTIME)}) \\ & - (\text{RNUTO}/100) - .0035)) \\ & + .59370 * \text{LOG(NL(-1)/NPOP(-1))} \end{aligned}$$

where NL is labour force, NPOP is population, F(QTIME) is a quadratic time trend prior to 80Q4 which is spliced to a linear time trend thereafter, NE is employment, and RNUTO is the deviation of the actual unemployment rate from the unemployment rate at the trend level of output.

The specification is such that an increase in employment accompanied by a decrease in the deviation of the actual from trend unemployment rate will have an impact on participation



rates that is roughly double the impact of an increase in employment resulting from a reduction in the trend unemployment rate. In the long-run an increase in the employment of 1 per cent would raise the participation rate by approximately .8 per cent, a 1 per cent increase in employment resulting from a decline in the trend unemployment rate would only increase the participation rate by .4 per cent.

### 3.8.10 MACE

In MACE the participation rate is a function of capacity utilization, the ratio of the natural to the actual rate of unemployment, the real wage in efficiency units, and a trend to capture increased female participation in the labour force. The estimated equation is:

$$(1) \quad (2)$$

$$\begin{aligned} J1P(Nl/Npl) = & .13754 * J2A(q/qsv) + .0072554 * (rnat/rnu) \\ & (3) \\ & + .0083480 * \text{sech}2((t-31)/16) \\ & (4) \\ & - .032904 * W/(5.9800 * LPI * Pa) \\ & + .004761 * Daib - .11190 \end{aligned}$$

where  $Nl$  is labour force,  $Npl$  is labour force population,  $q$  is gross output (at factor cost) of the non-energy sector,  $qsv$  is vintage-based synthetic supply,  $rnat$  is the natural rate of unemployment,  $rnu$  is the actual rate of unemployment,  $\text{sech}2$  represents the square of the trigonometric function arcsecant,  $t$  is a time trend,  $W$  is the wage rate,  $LPI$  is a labour productivity index,  $Pa$  is the absorption price index, and  $Daib$  is a dummy variable to account for effects of AIB equal to 1 in 1976 and -1 in 1978.

Term (1) is capacity utilization. High levels of capacity utilization cause the participation rate to increase. Term (2) captures the discouraged worker effect. Increased unemployment lowers the part rate unless it is associated with an increase in the natural rate. Term (4) shows that an increase in the real wage in efficiency units lowers the participation rate. Term (3) introduces the upward trend in the participation rate. The equation was estimated subject to the constraint that when the other driving variables have their normal values the rate of change in the participation rate is determined solely by the time trend.

### 3.8.11 SAM

In SAM the labour supply is assumed to result from the same intertemporal utility maximization decision by households as consumption. The estimated participation rate equation derived from this assumption is:

$$\begin{aligned}
 & \{ \hspace{10em} (1) \\
 \text{LS/NPOP} = & a_0 - ((a_1 * a_2 * ((1 + \text{NK})^{**a_3}) * ((\text{VT/PC} + \text{VHPVN})/\text{NPOP})) \\
 & /((1 - \text{RMTAX} + a_2 * (1 - \text{RATAXN}) * ((1 + \text{NK})^{**a_3} \\
 & * ((\text{WS/PS}) * (1 - \text{RNU}) + (\text{UIB/PS}) * \text{RNU} \\
 & \hspace{10em} \} \\
 & * ((1 - \text{RMTAX})^{**(- \text{QTXRFM})))))) \\
 & \hspace{10em} (2) \\
 & + a_6 * \text{LOG}(\text{PS/PC}) * (\text{VT/PC} + \text{VNSS})/\text{NPOP} \\
 & + a_7 * (\text{VHPV} - \text{VHPVN})/\text{NPOP}
 \end{aligned}$$

where LS is labour force, NPOP is the labour force source population, NK is the dependency rate, VT is financial wealth, PC is the consumption price index, VHPVN is human-wealth valued at steady state real wage, RMTAX is the marginal tax rate, RATAXN is the tax rate at the steady state real wage, WS is the steady state real wage, PS is the steady state consumption price index, RNU is the unemployment rate, UIB is the unemployment insurance benefit rate, QTXRFM is a dummy variable equal to 1 after 1972 for federal reform capturing taxation on unemployment insurance benefits after tax reform, VNSS is human wealth valued at steady state real wage and full employment, and VHPV is human wealth.

Term (1) represents the ratio of per capita real human and non-human wealth to the after-tax return from labour force participation (real wages and unemployment insurance benefits). Term (2) reflects the same disequilibrium influences discussed above in the consideration of SAM's consumption function.

## 3.9 Trade

### 3.9.1 CANDIDE 2.0

In CANDIDE 2.0 goods exports are disaggregated by industry and sometimes into commodities. Service receipts are disaggregated into freight and shipping, travel, income receipts, and other

service receipts. The export equations are generally demand functions including activity, relative price as measured by the ratio of the relevant U.S. price, converted to Canadian dollars, to the Canadian export price, exchange rate and cyclical variables, as explanatory variables. The activity variables are trade weighted OECD industrial production. In some of the service receipt equations foreign interest rates play a role. Export deflators are based on foreign prices translated into Canadian dollars.

Imports are also disaggregated by commodity. The equations contain activity variables related to industry output or final demand categories, and relative price terms. Service sector import equations exist for freight and shipping, travel, income payments, and other services. Import prices are also determined by converting world market prices into Canadian dollars using the exchange rate.

The main exchange rate equation in CANDIDE 2.0 is for the expected exchange rate. Under a flexible rate regime it is a function of the ratio of expected domestic prices to actual U.S. prices and the level of reserves. The expected price variable is based on past CPI inflation and past rates of growth of M1. It is the same variable as is used to explain wage behaviour.

The spot exchange rate (Can.\$ per U.S.\$) is a function of the expected exchange rate with a coefficient approximately equal to 1. When the exchange rate is floating short-term capital movements and the percentage change in the 3 month commercial paper rate cause the rate to vary from its expected value with short-term capital flows exerting upward pressure on the rate and increases in interest rates downward pressure.

### **3.9.2 TIM**

TIM has much detail in the trade sector. Exports and imports of automobiles and services are explicitly modelled. Exports are modelled depending on the features of the particular market. In markets where Canada must accept world prices, exports are a function of foreign activity variables and the prices of Canadian exports relative to some foreign prices adjusted by the exchange rate. In these markets export prices are defined as the corresponding foreign price translated into Canadian dollars. In markets where Canada is a price setter, exports are a function of activity variables and the price of Canadian exports relative to some indicator of the competitive price. For these goods, export prices are derived through cost mark-up within the input/output framework as are domestic prices.

Imports by category are modelled as a function of the relevant Canadian economic activity variable, and the price of imports relative to the relevant domestic price. Import prices are set on international markets.

The exchange rate in TIM is endogenized by a simulation rule that relates changes in the exchange rate to changes in the real current account balance. The coefficient in the relationship is a

reaction parameter that can be modified from one simulation to another.

### 3.9.3 RDX2

The export of goods part of the trade sector is highly aggregated in RDX2. There was little change from the original to the latest version. It consists of three equations, two for exports to the U.S. and one to the rest of the world. Exports of wheat and uranium, aircraft and parts are exogenous. Exports of motor vehicles and parts to the U.S is a function of a rising autopact variable times the sum of U.S. consumer expenditures and Canadian spending on durables.

Exports of goods to the United States (excluding uranium, aircraft and parts, and motor vehicles and parts) is a function of a 12 quarter export weighted moving average of U.S. final demand categories as an activity variable, the deviation of the current value of activity from the moving average, and a sixteen quarter moving average of the price of exports in the category to the price of non-farm business output in Canadian dollars. Exports of energy fuels to the U.S. which are included in the category are exogenous. The equation explains the residual.

Exports of goods to other countries (excluding wheat, uranium, and parts) is a function of a twelve quarter moving average of real world trade, the deviation of the current value of real world trade from the moving average, and the price of non-wheat exports to other countries relative to the price index of world trade.

The price indices for exports of goods (excluding wheat, uranium, aircraft and parts, and motor vehicles and parts) are functions of domestic prices and foreign prices (U.S. non-farm business output deflator and price index of world trade). The weight on foreign prices is only .17 in the equations of exports to other countries. The price of exports of motor vehicles is exogenous.

The import equations in the latest version of RDX2 were quite different from those in the original version of RDX2. They represent a much higher level of disaggregation with eight SITC categories for each of imports from the U.S. and from other countries. The equations were derived explicitly within a framework of traditional consumer theory of utility maximization in which the demand for imported goods was determined simultaneously with that for domestic goods. As a result the estimation of the equations involved the imposition of a number of constraints such as symmetry and homogeneity conditions through the utilization of an estimation procedure called joint least squares.

A simplified form of the equations for each SITC category is:

$$\begin{aligned} \text{LOG}(M12) = & a_0 + a_1 * \text{LOG}(PD) + a_2 * \text{LOG}(PM12) + a_3 * \text{LOG}(PM13) \\ & + a_4 * \text{LOG}(ACT) \end{aligned}$$

$$\begin{aligned} \text{LOG}(M13) = & b_0 + b_1 * \text{LOG}(PD) + b_2 * \text{LOG}(PM12) + b_3 * \text{LOG}(PM13) \\ & + b_4 * \text{LOG}(ACT) \end{aligned}$$

where M12 is imports in category from the U.S., M13 is imports in category from other countries, PD is the domestic price, PM12 is the price of imports in the category from the U.S., PM13 is the price of imports in the category from other countries, and ACT is weighted activity.

The homogeneity constraint requires that  $a_1 + a_2 + a_3 + a_4 = 0$  and  $b_1 + b_2 + b_3 + b_4 = 0$ .

The symmetry constraint means that  $a_3/W13 + a_4 = b_2/W12 + b_4$

where  $W13 = M13 * PM13/ACT$ , and  $W12 = M12 * PM12/ACT$ .

The actual functional form estimated for imports from the U.S. is:

$$\begin{aligned} \text{LOG}(M_{i12}/NPOPT) = & a_0 + (\text{seasonal dummies}) \\ & + a_1 * \text{LOG}((ACT\$i/NPOPT) * (1/PDi) \\ & * (UGPPD/UGPPA)) \\ & + a_2 * \text{LOG}((1 + ETARi) * (PMi12/PDi) * (UGPPD/UGPPA)) \\ & + a_3 * \text{LOG}((1 + ETARi) * (PMi13/PDi) * (UGPPD/UGPPA)) \end{aligned}$$

where  $M_{i12}$  is imports of category  $i$  from the U.S., NPOPT is total population,  $ACT\$i$  is a weighted activity variable appropriate to category  $i$ ,  $PDi$  is the price of domestic demand of category  $i$ ,  $UGPPA/UGPPD$  represents capacity utilization,  $ETARi$  is the equivalent ad valorem rate of duty collected on category  $i$ ,  $PMi12$  is the price of imports of category  $i$  from the U.S., and  $PMi13$  is the price of imports of category  $i$  from other countries.

Imports satisfy a larger proportion of demand the higher the level of capacity utilization.

Similar equations are in RDX2 for imports from other countries.

A problem posed by the sophistication of the theoretical model and the estimation constraints was that lags in the response of imports to activity and prices had to be ignored. The implied instantaneous reactions particularly to relative price changes were not very plausible.

Imports of motor vehicles from the U.S. was estimated outside of the rigid framework utilizing a much simpler specification in which imports are a function of relative prices, exports of motor vehicles to the U.S., and domestic consumption of North American motor vehicles.

Import prices in Canadian currency are tied to appropriate foreign prices through technical identities exhibiting a unitary elasticity with respect to exchange rate changes.

RDX2 has two exchange rate models - one for the fixed rate period and one for the floating. The equation for the floating rate period is:

$$\begin{aligned}
 & \text{(1)} \\
 \text{PFX} = & .21289 - .00473 * (\text{RS} - \text{RTB2}) + \{ .70327 * \text{PFXE} \\
 & \text{(2)} \\
 & * (\text{QPFX}) + .7644 * \text{PFXE} * (1 - \text{QPFX}) \} - 1.087260 \\
 & \text{(3)} \qquad \qquad \qquad \text{(4)} \\
 & * .00001 * (\text{UBAL}/\text{PFX} - \text{FXO}) + 1.192430 * .00001 \\
 & \text{(5)} \\
 & * \text{J1L}(\text{ULS}) + .00131 * \text{J8A}((1000 * \text{XOBES2}/\text{RCB2})/ \\
 & (\text{J8A}(\text{YGNE}/\text{RL})/\text{PFX})
 \end{aligned}$$

where PFX is the exchange rate in Can.\$ per U.S.\$, RS is the Canadian short-term government bond rate, RTB2 is the U.S. treasury bill rate, PFXE is the expected exchange rate, QPFX is a dummy variable equal to 1 during the first floating exchange rate period, UBAL is the basic balance, FXO is official demand for foreign exchange, ULS is the stock of outstanding short-term liabilities, XOBES2 is U.S. business product, RCB2 is the U.S. corporate bond rate, YGNE is Canadian GNE, and RL is the long-term Canadian bond rate.

The equation can be viewed as an inverted short-term capital flows equation. Term (1) captures the impact of the Canada-U.S. interest differential on short-term flows. Term (2) represents the effect of the expected exchange rate defined as a function of the exchange rate lagged one and two periods and the change in exchange reserves. Term (3) picks up demand for foreign

exchange

resulting from the basic balance and intervention. Term (4) is in many ways the most important variable in the equation. As long as a stock of outstanding liabilities exists there will be pressure on the exchange rate. This causes the exchange rate to move to equilibrate cumulative basic balance deficits. Term (5) is a type of portfolio demand variable measured by the ratio of U.S. wealth to Canadian. Wealth is calculated by capitalizing income using an interest rate.

### 3.9.4 CHASE

In the CHASE model exports of goods are disaggregated into chemicals and fertilizers, farm and fish products, forest products, lumber, metal and minerals, motor vehicles and parts to the U.S. and to the rest of the world, other manufactured goods, crude petroleum, and wheat. Services are disaggregated into freight and shipping, interest, dividends, travel receipts, and other. For exports of goods the general functional form of the equation is:

$$EX = f(\text{U.S. activity, U.S. capacity utilization, U.S. price in C\$} \\ \text{/Canadian export price})$$

The actual equations are estimated in logarithmic form.

In general, export prices are determined by foreign prices; however, some equations contain domestic prices reflecting the market power of exporters.

Imports of goods are decomposed into construction materials, crude petroleum, food, industrial materials, machinery and equipment, non-food consumption goods, miscellaneous fuels, and motor vehicles and parts to the U.S. and the rest of the world. Imports of motor vehicles and parts are also a function of exports to reflect the autopact. Imports of services are broken into interest, dividends, freight and shipping, travel payments, withholding tax and other services. The general function form of the equations is:

$$IM = f(\text{Canadian activity or income, Canadian capacity pressure,} \\ \text{U.S. price in C\$/Canadian export prices})$$

Again the equations are estimated in logarithmic form.

Import prices are based on foreign prices and the exchange rate.

The exchange rate in the CHASE model is a function of the interest differential, the percentage change in purchasing power parity, the change in the trade balance as a percentage of U.S. GNP, and the change in long-term capital flows as a percentage of U.S. GNP. The exact equation is:

$$\begin{aligned} J1P(EXRCA) = & -.37741 * J1D(R90CA - RCP90) - .83697 * J1D(R90CA - RCP90) \\ & + JW(J1P(JPPPCANUSCA(-1))) + 2.43699 * D77Q1 \\ & - 64.28810 * J4D(CBZCA(-1)/GNP(-1)) \\ & - 440.113 * J4D(FILCA(-1)/GNP(-1)) \end{aligned}$$

where EXRCA is the exchange rate in Can.\$ per U.S.\$, RC90CA is the 90 day finance paper rate, RCP90 is the U.S. interest rate on 90 day commercial paper, JPPPCANUSCA is an index of purchasing power parity based on Canadian and U.S. GNP deflators, D77Q1 is a dummy variable for the first quarter of 1977, CBZCA is the current account balance, GNP is U.S. GNP, and FILCA is long-term capital flows.

This is a standard enough exchange rate equation along the lines of that originally used in RDX2. An unusual feature is the use of the U.S. GNP to scale the current account balance and long-term capital flows. Canadian GNP has been more commonly utilized and probably makes more sense. The sum of the weights on the purchasing power parity term is 1.061. Thus if none of the other variables change, the exchange rate tends to follow a path consistent with purchasing power parity.

### 3.9.5 DRI

The DRI model has 10 export categories for goods, about the same level of disaggregation as CHASE: farm and fish products; petroleum and natural gas; newsprint and pulp; other forest products; iron, steel and aluminum; other metals; chemicals and products; machinery and tools; transportation equipment; and other exports. Service exports are disaggregated into five categories, one more than CHASE, the extra disaggregation coming from decomposing travel into U.S. and the rest of the world. The general specification of exports of goods equation is as a logarithmic function of foreign demand (trade weighted industrial production where appropriate) and domestic export prices relative to U.S. prices converted into Canadian dollars. Export prices are primarily a function of domestic industry prices determined within an input/output framework.

Imports of goods in the DRI model are also broken into ten categories: food, beverages and tobacco; petroleum; crude materials except petroleum; metals; chemicals; other fabricated materials; machinery; other equipment and tools; transportation equipment; and other imports. Service imports have the same breakdown as service exports. The general form of the import



equations is as a logarithmic function of import prices relative to domestic prices and the relevant measure of Canadian demand. In some cases Canadian demand is represented by an input/output weighted sum of final demand categories. Import prices are tied to U.S. wholesale prices through the exchange rate.

The exchange rate (Can.\$ per U.S.\$) in the DRI model depends on a weighted average of interest rate differentials, the basic balance plus net long-term capital flows, portfolio considerations as reflected in the ratio of U.S. wealth to Canadian (as measured by capitalized GNP), exchange rate expectations, and changes in foreign exchange reserves. The general functional form of the equation is:

$$\begin{aligned}
 \text{RXNS}\%\$@US = & f(\text{EXBP} - \text{MPP} + \text{BIPCAPNETL} + \text{BIPCRNTRSF} \\
 & + \text{BIPCAPNETLSF}, \text{RMFINCO90NS}, \text{US@RMGBS3NS}, \\
 & \text{US@RMEURO3NS}, \text{US@RMCDL}, \text{RXNS}\%\$@US(-i), \\
 & \text{JW}(((\text{US@GNP}/\text{US@RMGBLNS10CM} * \text{RXNS}\%\$@US/ \\
 & (\text{GNP}/\text{RMGBL10\&NS}))), \text{KLBFSNS}(-K), \\
 & \text{FXRES}/\text{FXRES}(-1))
 \end{aligned}$$

where  $i = 1,3$ ;  $K = 1,2$ ;

$\text{RXNS}\%\$@US$  is the exchange rate Can.\$ per U.S.\$,  
 $\text{EXBP} - \text{MPP}$  is the current account balance,  
 $\text{BIPCAPNETL}$  is net long-term capital flows,  
 $\text{BIPCRNTRSF}$  and  $\text{BIPCAPNETLSF}$  are seasonal factors for  
the current account and long-term capital flows,  
 $\text{RMFINCO90NS}$  is the 90 day paper rate,  
 $\text{US@RMGBS3NS}$  is the U.S. treasury bill rate,  
 $\text{US@RMGBS3NS}$  is the Eurodollar rate,  
 $\text{US@RMCDL}$  is the U.S. rate for certificates of deposit,  
 $\text{US@GNP}$  is U.S. GNP,  $\text{US@RMGBLNS}$  is the U.S. long-term  
government bond rate,  $\text{RMGBL10\&NS}$  is the Canadian long-  
term bond rate,  $\text{KLBFSNS}$  is the stock of outstanding  
short-term liabilities, and  $\text{FXRES}$  is changes in foreign  
exchange reserves.

Again the exchange rate equation is patterned on that in RDX2.

### 3.9.6 FOCUS

The trade sector of FOCUS is more highly aggregated than for the other quarterly models. Exports and imports of goods are disaggregated into automobiles, petroleum, and other. Export and imports of services are disaggregated into capital services and other. The automobile equations attempt to take into account the features of the auto pact by including domestic consumption in the export equation. Exports and imports of petroleum are exogenous.

The export of goods (excluding autos and petroleum to the U.S. (EXGOTH71) equation is as follows:

$$\text{LOG}(\text{EXGOTH71}) = 9.49183 + (\text{dummy variables for larger than usual}$$

$$\begin{aligned} & \text{wheat exports and other factors)} \\ & \quad (1) \quad (2) \\ & + \text{JW}(\text{LOG}(\text{JWTDEM})) + \text{JW}(\text{LOG}(\text{PEXGOTH}/\text{JWTDERCN})) \\ & \quad (3) \\ & + \text{JW}(\text{LOG}(\text{JWTDWPI})) \end{aligned}$$

where JWTDEM is a trade weighted index of industrial production, PEXGOTH is the deflator for other exports, JWTDERCN is the weighted U.S. world exchange rate index, and JWTDWPI is the weighted wholesale price index.

The equation is in logarithmic form so as to ensure constant elasticities. Term (1) represents foreign activity. The long-run elasticity of exports with respect to foreign activity is 1.22. Terms (2) and (3) capture relative price changes. Canadian prices in foreign currency and foreign wholesale prices are entered separately. The former has a long-run elasticity of -1.24 and the latter 1.60.

The change in the price of exports of other goods is a function of changes in the private GNP deflator and a weighted average of world prices with weights of 77 per cent and 23 per cent respectively. Canada is thus treated as a price-maker in export markets.

The equation for imports of other goods (MGOTH71) is:

$$\begin{aligned} & \quad (1) \\ \text{LOG}(\text{MGOTH71}) = & -.734601 + .588062 * \text{LOG}(\text{UCAPIND}) \\ & + (\text{assorted dummy variables}) - .901795 \\ & \quad (2) \\ & * \text{LOG}(\text{PEX@US} * \text{ZPMTAX}) \\ & \quad (3) \\ & + \text{JW}(\text{LOG}(\text{MWTDEM})) \\ & \quad (4) \end{aligned}$$

$$\begin{aligned}
& + JW(\text{LOG}(\text{RXNS}\%\$@\text{US} * \text{DMYFL2A})) \\
& \quad (5) \\
& + JW(\text{LOG}(\text{PMDOM}))
\end{aligned}$$

where UCAPIND is the capacity utilization rate defined in terms of private business GNP, PEX@US is the U.S. export price index, ZPMTAX is an adjustment for domestic sales tax rates, MWTDEM is a weighted domestic demand term, RXNS%\$@US is the exchange rate (C\$/US\$), DMYFL2A is a dummy equal to 1 after the floating of the Canadian dollar, and PMDOM is a proxy for the domestic competitive price.

Term (1) captures the phenomenon of imports as a residual source of supply. Term (2) represents the price of U.S. exports (Canadian imports) adjusted for indirect taxes. This in comparison with term (4) for the exchange rate when floating and term (5) for the domestic competitive price constitutes the relative price effect. The elasticity on the exchange rate of 1.01 and on domestic competitive prices of .91 are close to that on the U.S. export price. Term (3) captures weighted domestic demand. Its long-run coefficient is 1.

The price deflator for other goods is a function of the exchange rate and U.S. export prices reflecting price taking behaviour.

Exports of services excluding capital services are a function of U.S. GNP, relative prices and the interest rate on medium-term government bonds. Exports of capital services respond to the stock of Canadian direct investment abroad, U.S. interest rates, the exchange rate, and U.S. capacity utilization.

Imports of services excluding capital services are determined by Canadian real GNP, relative prices, total imports, the stock of U.S. direct investment, the 90 day paper rate and the change in the exchange rate. Imports of capital are a function of the stock of provincial/municipal and corporate bonds held abroad, current and lagged changes in Canadian/U.S. interest rate differentials, and the U.S. interest rate on long bonds.

The exchange rate in FOCUS is determined as the price that clears the market for foreign exchange. This is made possible by the inclusion of equations or exogenous values for all of the components of the current and capital account of the balance of payments including an equation for short-term capital flows. This short-term capital flow equation is highly sensitive to changes in the exchange rate and tends to stabilize the foreign exchange market. Additional stability can be introduced through the imposition of a rule for official intervention which is optional with a parameter for the degree of intervention.

### 3.9.7 MTFM

MTFM has the largest trade sector as measured by the total number of equations of any of the quarterly models. It has a few less stochastic equations than QFS, however, and less disaggregation of export and import categories. Exports are divided into 15 categories for goods and 6 categories for services. The categories for goods are: wheat and barley; other crude agricultural products; coal and other crude bituminous materials; crude petroleum; natural gas; crude materials and minerals excluding mineral fuels; food, beverages and tobacco; lumber and other wood fabricated products; pulp and paper products; primary metals; automobiles and parts to the United States and to the rest of the world; chemicals; machinery and equipment; and other manufactured goods.

Exports of goods are a logarithmic function of 1) foreign demand as measured by a trade weighted average of output or expenditure, 2) the export price or domestic cost of production relative to foreign prices or costs, and 3) capacity utilization. For goods where Canada is a price taker, profitability is a key factor in the export equation; where Canada is a price setter it is a relative price that counts. Some export prices are determined primarily by domestic industry prices, others by world prices and the exchange rate. Foreign and domestic demand conditions also enter in some of the equations as explanatory variables.

Exports of services are disaggregated into interest and dividends, travel, freight and shipping, and other. Explanatory variables are domestic and foreign expenditure aggregates, prices and the exchange rate.

Imports of goods are divided into 12 categories: crude agricultural products; food, beverages and tobacco; crude oil; crude metals and minerals excluding mineral fuels; primary metals; textiles, knitting and clothing; chemicals; petroleum and coal products; automobiles and parts from the U.S. and from other countries; machinery and equipment; and other manufactured goods.

The imports of goods equations are based on an import share approach. According to this approach, the long-run share of imports in domestic demand is a function of: 1) the tariff-adjusted import price for the foreign production cost relative to the domestic price or cost (MTFM and SAM are the only models since RDX2 to make an adjustment for the tariff); and 2) the capacity utilization rate in manufacturing. A time trend is also included in some of the equations. In the short run, imports respond less than one-for-one to changes in domestic demand.

Domestic demand variables are specified to correspond as closely as possible to the import category. They include both intermediate and final demand and are weighted using input/output weights. Sometimes permanent income is utilized as a proxy for demand. Import prices are generally specified as a function of the exchange rate and lagged U.S. prices. Demand variables are included in some of the import price equations.

The equations for imports of services include such variables as domestic or foreign expenditures, relative prices, and interest rates.

The MTFM exchange rate equation is in effect an inverted capital flows equation. A difference from many other equations of this type is that it does not include a term for the lagged stock of liabilities. The equation is:

$$\begin{aligned} \text{PFX} = & .000608 + .29872 * (\text{FOMNET} - \text{FOMLIAB} - \text{FSDR} - \text{BBAL})/(\text{XM} + \text{MM}) \\ & - .00431 * \text{J1D}(\text{RCP90} - \text{USRCP90}) - .00431 * \text{J1D}(\text{RCP90}(-1)) \\ & - \text{USRCP90}(-1)) + 1.06158 * \text{PFX}(-1) + (1 - 1.606158) * \text{PFX}(-2) \\ & + .01758 * \text{DPFX} \end{aligned}$$

where PFX is the exchange rate Can.\$ per U.S.\$,  
(FOMNET - FOMLIAB - FSDR) is change in exchange reserves,  
BBAL is the basic balance, XM is nominal merchandise exports,  
MM is nominal merchandise imports,  
RCP90 is the 90 day paper rate, USRCP90 is the U.S. 90 day  
paper rate, and DPFX is a dummy variable reflecting  
downward pressure on the Canadian dollar from 76Q4 to 78Q4.

The lagged exchange rate terms in the equation are interpreted as representing expectations.

### 3.9.8 QFS

QFS is the quarterly model with the largest number of stochastic equations in the trade sector. The QFS trade sector differs from those of DRI and MTFM in that it does not make use of input/output coefficients in calculating demand variables.

The export of goods categories included in QFS are: wheat and flour; other agricultural products; lumber; pulp and paper; metals and minerals; other fabricated materials; passenger cars; commercial vehicles; automotive parts; machinery and equipment; other manufactured products; crude petroleum; and natural gas. The export goods volumes are mostly demand equations. The equations estimated in logarithmic form are a function of indexes of world real GDP weighted by Canadian exports, and of relative prices expressed as the ratio of the relevant export price and the product of a similar U.S. price and the rate of exchange. Export prices for goods are supply equations based on a cost mark-up on domestic wage and non-wage costs. The mark-up is influenced in the short-run by variations in the capacity utilization rate for manufacturing or the ratio of actual to potential output.

Exports of services are disaggregated to a greater extent than the other quarterly models. The extra categories result from decomposing travel receipts from the U.S. and the rest of the world and from breaking out the miscellaneous component of other services, which represents short-term interest receipts. Imports of services also exhibit the same higher degree of disaggregation.

Exports of services in current dollars are a function of various measures of bond holdings, U.S. corporate profits, the stock of non-bank short-term foreign currency assets, and Canadian chartered banks' total foreign currency assets.

The merchandise import categories in QFS include: food; other consumer goods from the U.S., and from the rest of the world; industrial materials; commercial vehicles from the U.S. automotive parts from the U.S.; passenger cars from the U.S.; and crude petroleum. The import of goods equations are logarithmic demand functions of domestic permanent income for consumer categories and investment for producer categories, relative prices measured by the ratio of the relevant import price to a weighted average of domestic costs, and demand pressure proxied by the inverse of the capacity utilization rate for manufacturing. Import prices reflect supply factors and are a function of U.S. wages and material costs or final demand prices in the U.S. converted into Canadian dollars.

Imports of services are a function of the domestic stock of long-term bonds, domestic after-tax corporate profits, the exchange rate, and various interest rates. The QFS equation for the foreign exchange rate (PFX) expressed in Can.\$ per U.S.\$ can be summarized as:

$$\begin{aligned}
 & \text{(1)} \\
 \text{LOG(PFX)} = & -.00180 - .00519 * (\text{RFCP90} - \text{RFCP902}) \\
 & \text{(2)} \\
 & + (1 - .79758 + .11987 * \text{Q764}) * \text{JW("ULC")}/\text{JW("ULCUS")} \\
 & \text{(3)} \\
 & + (.79758 - .11987 * \text{Q764}) * \text{LOG(PFX(-1))} \\
 & \text{(4)} \\
 & - .06399 * (\text{PXG(-1)}/\text{PMGUS(-1)}) - \text{J2L(J8A(PXG}/\text{PMGUS))} \\
 & \text{(5)} \\
 & - .31711 * (\text{UBALG} - \text{J1L(J8A(UBALG))})/\text{GNE} \\
 & \text{(6)} \\
 & - .38336 + (\text{UBALLTCA} + \text{UBALS})/\text{GNE}
 \end{aligned}$$

where RFC90 is the 90 day commercial paper rate, RCP902 is the U.S. 90 day commercial paper rate, Q764 is a dummy equal to

.5 in 1976Q4 and 1 thereafter, ULC is unit labour costs, ULCUS is U.S. unit labour costs, PXG is the price deflator for exports of goods and PMGUS is the price deflator for imports from the U.S., UBALG is the current account balance on goods, UBALLTCA is the balance on long-term capital flows, and UBALS is the balance on services.

Term (1) is the interest rate differential. Term (2) is the relative unit labour cost variable that injects a tendency towards purchasing power parity. This tendency dominates exchange rate determination in the longer-run. Term (4) represents the deviation of the ratio of the price of exports of goods to the price of imports of goods from the U.S. from its 8 quarter moving average. Term (5) is the deviation relative to GNE on the goods balance from an 8 quarter moving average. Term (6) is the balance on services and long-term capital flows relative to GNP.

### **3.9.9 RDXF**

Exports of goods in RDXF are divided into eleven categories: motor vehicles and parts to the U.S. and to the rest of the world; farm and fish products; wheat; metals and minerals; chemicals and fertilizers; lumber; forest products; petroleum; natural gas; and other manufactured goods. The export equations are loglinear demand functions. The most important explanatory variables are U.S. activity, usually represented as real GNP, and the export price relative to the appropriate U.S. price converted into Canadian dollars. Also included are foreign overseas activity variables calculated as the weighted average of the indices of industrial production in Japan and Europe with the weights particular to the export category. In some of the equations such as exports of motor vehicles, farm and fish products, or other manufactured goods, consumption or investment categories replace U.S. GNP as the activity variable. Two of the equations for chemicals and fertilizers have supply terms.

Exports of crude petroleum, natural gas, wheat, and motor vehicles to the rest of the world are exogenous. Exports and imports of motor vehicles from the U.S. are interdependent reflecting the autopact.

Export prices are generally specified as loglinear functions of related U.S. prices converted to Canadian dollars. Often the dependent variable enters with a lag. The equation for exports of other goods is a function of Canadian and U.S. costs. The elasticity of export deflators with respect to the exchange rate is approximately one in the long-run.

Exports of services are divided into four categories: travel; freight and shipping; other services; and interest and dividends. Travel is a function of U.S. consumption of services and relative prices. Freight and shipping responds to the nominal exports of goods. Other services are determined by the U.S. treasury bill rate and U.S. nominal GNP. Interest and dividends depend on

Canadian direct investment, foreign exchange reserves and U.S. long-term interest rates.

Merchandise imports in RDXF are disaggregated into nine categories: industrial materials; producers equipment; motor vehicles and parts from the U.S.; motor vehicles and parts from other countries; construction materials; other consumer goods excluding food; crude petroleum; and other fuels.

The import equations are based on consumer demand theory which involves a choice between domestic and imported goods so as to maximize utility given a budget constraint. The theory underlying the equations for imports of consumer goods is different, and is based on the demand for factors of production. Imports of producers equipment and construction materials is related to investment demand and capacity utilization. The general form of the import equations is:

$$\text{LOG}(M_i) = a_0 * \text{LOG}(P_{Mi}/P_i) + a_1 * \text{LOG}(U_i) + a_2 * \text{LOG}(\text{CAPU})$$

where  $M_i$  is imports of category  $i$ ,  $P_{Mi}/P_i$  is the price of imports of category  $i$  relative to price of domestic demand or activity, and CAPU is an aggregate capacity utilization rate calculated as the ratio of real gross private business product to trend output and is only utilized in equations for imports of producer goods

Import prices are determined consistent with the small country assumption as a function of U.S. costs translated into Canadian dollars. In at least two cases a flexible mark-up dependent on the level of U.S. capacity utilization is applied to U.S. costs.

Imports of services are disaggregated into seven categories: interest payments to foreigners; dividend payments; travel payments; freight and shipping; other services; miscellaneous payments; and withholding taxes payable by non-residents. Factors influencing these service imports include the exchange rate, interest rates, foreign liabilities, activity and relative prices.

The RDXF equation for the spot exchange rate (PFX) in Can.\$ per U.S.\$ can be summarized as:

$$\begin{aligned} \text{J1D}(\text{LOG}(\text{PFX})) = & \text{(1)} \\ & .00701 - .43078 * (\text{R90} - \text{RCP2})/100 \\ & \text{(2)} \\ & - .20616 * \text{J1L}(\text{J1D}(\text{XG\$} - \text{MG\$})/\text{YGNE}) \\ & \text{(3)} \\ & - 1.40894 * \text{J1L}(\text{J1D}(\text{FIL})/\text{YGNE}) \\ & \text{(4)} \\ & - .82778 * (\text{UBAL}/\text{YGNE}) \\ & \text{(5)} \end{aligned}$$



$$+ .23671 * (\text{LOG}(J4A(\text{PGNE})/J4A(\text{PGNPUS}))$$

$$- \text{LOG}(J1L(\text{PFX})) + (\text{other time related variables})$$

where R90 is the Canadian 90 day commercial paper rate, RCP2 is the U.S. commercial paper rate, XG\$ is exports of goods in current dollars, MG\$ is imports of goods in current dollars, YGNE is nominal GNE, PGNE is the price deflator for GNE, and PGNPUS is the price deflator for U.S. GNP.

Term (1) represents the interest differential and term (2) the merchandise trade balance as a percentage of GNP. Term (3) captures the depressing effect on the exchange rate of the balance on long-term capital flows as a percentage of GNP. Term (4) captures the impact of the basic balance. Term (5) is for purchasing power parity. RDXF like QFS basically reflects purchasing power parity in the long-run.

### 3.9.10 MACE

In MACE exports are disaggregated into exports of primary energy, non-energy exports, exports of cars to the U.S., and interest and dividend receipts. Energy exports are determined in the large energy block of the model. Exports of cars to the U.S. are a function of relative prices as defined by the ratio of the price of non-energy exports to the price of U.S. absorption in Canadian dollars, and of a complex time trend reflecting the autopact. The equation for interest and dividend receipts (Xid) is as follows:

$$\text{Xid} = .12378 + .015720 * \text{Pfx} * \text{rtb2} * \text{J2A}(\text{Ures})$$

$$+ .000024378 * \text{Pfx} * \text{rm2} * \text{a2}$$

where Pfx is the price of foreign exchange C\$ per U.S.\$, rtb2 is the U.S. treasury bill rate, Ures is foreign exchange reserves, rm2 is the average yield on U.S. government bonds, and a2 is U.S. absorption.

Term (1) captures interest receipts on foreign exchange reserves and term (2) the return on Canadian owned assets in the United States.

Non-energy exports excluding cars to the U.S. and interest and dividends are explained by the following equation:

$$(1)$$

$$\begin{aligned} \text{LOG}(xne - xcar2 - xid/pxne) = & -6.0768 + 1.1661 * \text{LOG}(yw) \\ & (2) \\ & - .43028 * \text{LOG}(J3A(pxne/(pfx * pwxg))) \\ & (3) \\ & + .35060 * \text{LOG}(J2A(Kinv)/Kinv*) \end{aligned}$$

where xne is non-energy exports, xcar2 is exports of cars to the U.S., xid is receipts of interest and dividends, yw is real output in major OECD countries, pxne is the price of non-energy exports, pfx is the exchange rate C\$ per U.S.\$, pwxg is the price of world traded goods, Kinv is the stock of inventories, and Kinv\* is the desired stock of inventories.

Term (1) represents foreign demand. Term (2) allows for an influence of relative price. Term (3) is a supply variable. When inventories are high relative to desired levels, there is a greater tendency to seek foreign markets.

The price of non-energy exports is a function of the price of gross domestic output, the exchange rate and the price of world traded goods. The actual equation is:

$$\begin{aligned} J1P(pxne) = & -.0078226 + .60456 * J1P(pq) + .39544 * J1P(pfx) \\ & + .35944 * J1P(pwxg) \end{aligned}$$

where the variables are as described above, and pq is the price of domestic output.

The coefficients on the exchange rate and the price of world traded goods are constrained to be the same. The sum of the value of this coefficient and the coefficient on the price of domestic output are constrained to be unity.

Non-energy imports excluding cars from the U.S., which are explained by relative prices and a complex increasing trend of North American absorption, are a function of aggregate supply and the price of non-energy imports relative to the price of domestic output. The specific equation is:

$$\text{LOG}(mne - mcar2) = -1.5672 + \text{LOG}(qsv) - 1.3474 * \text{LOG}(J3A(pme/pq))$$

where mne is non-energy imports, mcar2 is imports of cars from the U.S., qsv is vintage-based synthetic supply, pme is the price of non-energy imports, and pq is the price of domestic output.

Non-energy imports excluding cars are estimated as a proportion of supply as suggested by

theory. The price term is the price of non-energy imports relative to the price of domestic output.

The price of non-energy imports is defined to be equal to the exchange rate times the OECD real output deflator.

Mace has a number of exchange rate regimes. The standard regime is the portfolio model with limited intervention. Under this regime the authorities are assumed to intervene offsetting any excess demand (or supply) for foreign exchange from the current and capital accounts. Using the exchange fund, but allowing the exchange rate to depreciate (appreciate) by .7 cents for every \$100 million change in reserves.

The other regimes are the fixed rate under which the exchange rate is exogenous, nominal interest parity, and purchasing power parity. Under nominal interest parity the exchange rate is set so as to equate 3-to-5 year term interest rates in Canada and the U.S. given exchange expectations based on weak purchasing power parity. Under purchasing power parity the exchange rate is set to equilibrate the price of Canadian output in U.S. dollars with the price of U.S. absorption at their historic ratio.

### 3.9.11 SAM

SAM has only one export and one import equation for non-energy goods. Energy trade is exogenous. Non-energy exports are modelled as:

$$\begin{aligned} \text{LOG(XNEID)} = & \text{(1)} \\ & -2.3587 - .3026 * \text{LOG(PXNEID)/(PFX * PW)} \\ & \text{(2)} \quad \text{(3)} \\ & + .0362 * \text{DUMCAR} + 1.3716 * \text{LOG(UGPBSS)} \\ & \text{(4)} \\ & + .9976 * \text{LOG(YW/UGPBSS)} - .0370 \end{aligned}$$

where XNEID is exports net of energy, PXNEID is the price of exports net of energy, PFX is the exchange rate C\$ per U.S.\$, PW is the world price index, DUMCAR is a dummy variable

for the autopact, UGPBSS is total real private sector output in the steady state, and YW is world real income.

Term (1) is the relative price term. Term (2) captures the impact of the autopact in raising exports of cars. Term (3) is a domestic supply term. Its coefficient is constrained to be the same as estimated in the import equation so that with steady state growth the current account balance does not exhibit a tendency to improve or deteriorate due to real trade flows. Term (4) allows for short-run disequilibrium effects resulting from change in world demand.

The price of non-energy exports is specified as:

$$\begin{aligned} & \text{(1)} \\ \text{J1D}(\text{LOG}(\text{PXNEID})) &= .5954 * \text{J1D}(\text{LOG}(\text{PD})) \\ & \text{(2)} \\ & + (1 - .5954) * \text{J1D}(\text{LOG}(\text{PFX} * \text{PW})) \\ & \text{(3)} \\ & + .1262 * \text{J1D}(\text{LOG}(\text{PCW2}/\text{PW})) \end{aligned}$$

where PXNEID is the price of non-energy exports, PFX is the exchange rate C\$ per U.S.\$, PW is the world price index, and PCW2 is the Economist's World Commodity Price index.

Term (1) represents a weighted average price of domestic output in the domestic market and term (2) the price of world output in domestic currency units. Term (3) allows for the differential impact of commodity prices on export prices.

The equation for non-energy imports is:

$$\begin{aligned} & \text{(1)} \\ \text{LOG}(\text{MNEID}) &= -5.6501 + .7709 * \text{LOG}(\text{P}/(\text{PMEID} * (1 + \text{RTAR}))) \\ & \text{(2)} \qquad \qquad \text{(3)} \\ & + 1.3716 * \text{LOG}(\text{UGPBSS}) + .7971 * (\text{CAPU} - \text{CAPUS}) \\ & \text{(4)} \\ & + .0152 * \text{DUMCAR} \end{aligned}$$

where MNEID is non-energy imports of goods, P is the domestic production selling price index, RTAR is the tariff rate, UGPBSS is steady state private output, CAPU is capacity utilization of capital, CAPUS is the planned level of capacity utilization before demand shocks, and DUMCAR is an autopact dummy variable.

Term (1) represents the price of domestic output relative to import prices after the tariff. Term (2) is steady state output. The coefficient is that which was imposed on the export equation. Term (3) permits an impact of capacity utilization on imports.

The price of non-energy goods is determined by the following technical relationship:

$$PMNEID = PRELM * PFX * PW$$

where PRELM is the relative price of imports.

The service account in SAM is limited to interest and dividend flows. While there are no behavioural equations for the flows, behaviour is specified in the stock dimension with explicit identities linking the interest and profit flows to the stocks held.

The exchange rate (PFX) in SAM is determined in the market for net foreign assets in accordance with the Branson portfolio approach, although its value is influenced by real variables in the simultaneous solution of the model in simulation. Net foreign assets are one of four financial assets -- money, bonds, and equity being the other three -- that are interdependent in a constrained portfolio model. The net foreign asset demand equation specifies the desired share of financial wealth in this form as dependent on three interest rate differentials (foreign assets relative to government bonds, equity, and money, respectively). It can be written as follows:

$$PFX * F/V = a_0 + a_1 * (RFE - RGE) + a_2 * (RFE - REQE) + a_3 * RFE$$

where F is net foreign assets, V is financial wealth, RFE is the expected after-tax rate of return on net foreign assets, RGE is the expected after-tax return on government bonds, and REQE is the expected after-tax rate of return on equities.

There is also an asset valuation or interest parity condition that states that the return to foreign assets is the exogenous foreign rate, RUS modified by the expectations of capital gains or losses in the exchange market. It is specified as:

$$\text{LOG}(PFX) = \text{LOG}(PFXE) + RUS - RFE$$

Finally, there is an equation for the expected exchange rate. PFXE, the one period ahead expected exchange rate is modelled as:

$$\begin{aligned}
 & (1) \\
 J1D(\text{LOG}(\text{PFXE})) &= b_0 * J1D * (\text{LOG}(\text{PFX} * \text{PUS}/\text{PC})) \\
 & (2) \\
 & + b_1 * J1L(J1D(\text{LOG}(\text{PFX} * \text{PUS}/\text{PC}))) \\
 & (3) \\
 & + b_2 * J1D(\text{LOG}(\text{PC}/\text{PUS})) \\
 & (4) \\
 & + b_3 * J2A(\text{LOG}(\text{PFX}/J1L(\text{PFXE})))
 \end{aligned}$$

where PFXE is the expected exchange rate, PUS is the U.S. price index, and PC is the consumer price index

For purposes of estimation, PFXE is measured as the one-year forward rate. The first two terms of the equation allow for the impact of the real exchange rate. The third term captures relative purchasing power parity. The fourth term adjusts for recent errors in expectations.

The three equations for net foreign asset demand, interest parity, and the expected exchange rate, given the quantity of net foreign assets as determined by the supply equation linked to the current account balance, constitutes a three equation system that determines the exchange rate (PFX), the expected exchange rate (PFXE), and the return on net foreign assets (RFE). The system is not closed, but depends on other endogenous and exogenous variables in the model.

### 3.10 Government

#### 3.10.1 CANDIDE 2.0

Government revenue and expenditures in CANDIDE are disaggregated by level of government.

CANDIDE has a detailed personal income tax calculator as in RDX2 which utilizes information on income distribution to calculate personal tax liabilities. CANDIDE also has federal and provincial corporate tax equations and equations for most indirect tax categories. The general form is as a rate-base calculation.

The Unemployment Insurance account in CANDIDE is modelled structurally with attention to detail. Canada and Quebec Pension Plan revenues are based on exogenous contribution rates and maximum taxable earnings. Benefits are indexed in the model and respond to demographic

variables.

EPF and equalization are exogenous. Canada Assistance Plan transfers to provinces are a function of provincial spending on welfare.

Government current purchases of goods and services are disaggregated by level of government. Behavioural equations, which can be characterized as reaction functions, are included for most categories of current and capital spending. Employment is also determined by reaction functions in most cases. Wages are a function of expected inflation and, for the federal government, the prime age male employment rate.

Federal government interest payments are a function of interest rates times the stock of public debt by category. The fact that the coefficient is significantly greater than one is a source of instability. Provincial, municipal and local debt service is also a rate base calculation.

### **3.10.2 TIM**

Government revenues and spending in TIM are also broken down into federal, provincial, municipal, hospitals, and pension plans.

The personal income tax is determined by a set of structural equations rather than a personal tax calculator. Corporate tax equations are included for both federal and provincial government. Equations are included for all of the major indirect tax categories by level of government.

The UI account and C/QPP are modelled in TIM. Interest on the public debt is calculated by level of government by multiplying the appropriate interest rates times the stock of debt or accumulated deficits.

EPF is modelled as the sum of its education and medicare components which are a function of a rate times school enrollment and population respectively. CAP transfers are a function of provincial spending on welfare and an exogenous rate. Equalization is specified as a rate times federal income tax receipts.

Government current expenditures are disaggregated by level of government. The approach to modelling them is similar to that in CANDIDE.

### **3.10.3 RDX2**

RDX2 has revenues and expenditures disaggregated by levels of government.

Personal income tax in RDX2 is explained by a detailed 191 equation income tax calculator that takes information on number of taxpayers, exemptions, deductions and income, and using

displaced lognormal distributions and the federal tax table generates federal and provincial income tax.

Corporate income taxes and indirect taxes are modelled by rate-base calculations.

The revenues and expenditures of the UI account are explained by structural equations, but the revenues and expenditures of the C/QPP are exogenous.

Interest on the federal public debt depends on interest rates and the stock of three categories of government debt. Interest on the provincial-local public debt is exogenous.

The various categories of government expenditures on goods and services in real terms are determined by reaction functions. Explanatory variables include such variables as income per capita, relative prices, the unemployment rate, population, and interest rates. Government wages are a function of private sector wages.

### **3.10.4 CHASE**

Government revenue and expenditure detail is generated in the CHASE model. Total direct personal taxes are calculated by subtracting deductions and exemptions from assessed income to yield total taxable income and then dividing the total by the number of taxpayers to get an estimate of the income of a representative taxpayer. The average tax liability is then calculated using the federal tax table and multiplied by the number of taxpayers.

Corporate taxes are a function of the tax rate and corporate profits, the latter being adjusted to take account of capital consumption allowances.

Indirect taxes are broken down by federal and provincial levels of government. Categories modelled include manufacturers' sales tax, custom duties, excise duties, and oil export and other NEP taxes at the federal level and retail sales tax and municipal indirect taxes at the provincial-municipal level.

UI revenue depends on maximum insurable earnings, the contribution rate, employment, and the ratio of wages to maximum insurable earnings. UI benefits are a function of unemployment and the benefit rate. CPP revenues depend on wage income. Benefits and other expenditures are exogenous.

Interest on the federal public debt is a function of interest rates times the stock of various categories of public debt including public sector pension fund balances. Interest payments of other levels of government is exogenous.



Federal transfers to provinces are exogenous.

Government current expenditures are decomposed by level of government and are exogenous in real terms.

### **3.10.5 DRI**

Personal income tax in the DRI model is calculated by applying a measure of the effective basic federal income tax rate to an estimate of the base.

Federal and provincial corporate taxes are specified as a function of the aggregate corporate tax rate times the corporate income tax base.

Indirect taxes at the federal level are divided into three categories: federal sales tax; customs duties; and other. At the provincial level there are two categories: retail sales tax and other. Equations are rate-base calculations.

UI contributions depend on the product of employment, the UI maximum insurable earnings, and the total contribution rate. UI benefits are a function of maximum weekly benefits times the number of unemployed.

CPP contributions are a function of the employer and employee contribution rate, non-exempt pensionable earnings, and the level of employment. CPP benefits are determined by the product of the maximum monthly indexed benefit and the population over 65.

Interest on the public debt for both levels of government is expressed as a function of an average of short and longer term interest rates times a synthetic cumulated deficit.

Federal transfers to provinces are exogenous.

Federal and provincial current expenditures can be specified to be exogenous in either real or nominal terms. Current provincial expenditures can also be made endogenous using a reaction function.

### **3.10.6 FOCUS**

Government revenues and expenditures in FOCUS are separated by level of government.

Federal and provincial personal income taxes are calculated in a number of steps. Median gross income and average personal exemptions are utilized to determine average taxable income. Federal assessed taxes are determined from average taxable income and the federal marginal tax rate with an adjustment for indexing. Collections depend on assessments with a lag. Provincial

income taxes also are based on assessments.

Federal and provincial corporate income taxes are a function of statutory corporation tax rates and the estimated tax base, adjusted for investment tax credits.

The federal sales tax is modelled stochastically as a function of the rates times the appropriate bases. Import duties are specified in a technical relationship as the effective average duty rate times other merchandise imports. Total federal indirect taxes are estimated as the sum of federal sales taxes and customs duties and a proportion of total other indirect taxes.

Local indirect taxes are adjusted in the first quarter depending on population size and past inflation and in other quarters move at the growth of population. Provincial retail sales taxes are included in an other indirect tax category which also encompasses federal indirect taxes other than the sales tax.

UI contributions are expressed as a function of contribution rates, maximum insurable earnings, and average wages and salaries. Benefits are determined by maximum weekly benefits and the number of unemployed.

Contributions to C/QPP are a function of the contribution rate, maximum annual earnings and the average wage. Benefits are exogenous.

Interest on the federal public debt is expressed in first difference form as a function of the deficit less the monetary base times the interest rate on long-term government bonds. The equations for interest on the public debt of other levels of government are similar.

Net federal transfers to other levels of government are determined by a simulation rule requiring them to grow at the same rate as a three year moving average of GNP. This rule reflects the EPF arrangements.

Federal, provincial and other levels of government expenditures are exogenous in real terms.

### **3.10.7 MTFM**

Government revenue and expenditure in MTFM are separated into federal, provincial-local-hospital, and pension plans.

The personal income tax is determined by a number of equations which calculate the number of equations of taxfilers, exemptions, deductions and taxable income. The key equation relates basic federal tax to taxable income per taxfiler divided by the indexing factor.

Corporate tax equations are rate-base calculations with capital cost allowances by industry, and other institutional features of the tax system taken into account. Most categories of indirect

taxes including energy taxes are modelled.

UI and C/QPP revenues and expenditures are approximated by linear equations reflecting the contribution and benefit structures. Interest payments on the federal debt are a function of interest rates and debt stock. Interest payments of other levels of government are exogenous

Total federal transfers to regional governments are exogenous except for the oil export tax.

Government expenditure for the three levels of government (federal, PLH, and pensions) are exogenous in real terms.

### **3.10.8 QFS**

As befits a model developed at the Department of Finance, QFS has the largest government sector of any of the quarterly models. Much detail is included on revenues and expenditures and by level of government - federal, PLH, and pension plans.

The personal income tax equations depend on the traditional variables of taxfilers, exemptions, deductions and taxable income. The equations are estimated using annual data.

Corporate tax equations are fairly straightforward rate-base calculations as are the indirect tax equations.

UI and C/QPP revenues and expenditures are modelled based on contribution rates and earnings maximums. Changes in UI benefits are tied by a simulation rule to changes in unemployment.

Interest payments on the federal debt are endogenized through a simulation rule that relates the change in interest rates from the control to changes in effective interest costs on outstanding debt, and changes in the debt stock itself.

Federal transfers to other levels of government are exogenous.

Government expenditure for the three levels of government are exogenous in nominal terms, but can also be made exogenous in real terms.

### **3.10.9 RDXF**

RDXF has a reasonable amount of detail on government revenue and expenditures.

In RDXF personal income tax is modelled based on the tax liability of a representative taxpayer earning the average taxable income. The calculation makes use of data on the number of taxfilers, exemptions, and deductions, and various types of income each with its own

assessment ratio.

Federal and provincial corporate income tax are calculated by scaling taxable corporate profits by a weighted average corporate tax rate. Taxable profits are approximated by applying various deductions and exemptions to national accounts before-tax profits.

Indirect taxes in RDXF are disaggregated into six taxes at the federal level and three at the provincial-municipal level. The federal indirect taxes are: the manufacturers' sales tax; customs duties; the oil export tax; the excise tax on gasoline; excise duties; and miscellaneous excise duties. Provincial-municipal indirect taxes are: the retail sales tax; other provincial indirect taxes; and municipal indirect taxes (mainly property taxes). The equations are expressed in terms of an appropriate tax base multiplied by a weighted average rate.

The unemployment insurance account is explained by one equation for revenues, which depends on the contribution rate, the number of unemployed, and maximum insurable earnings, and by an equation for benefits, which are a function of the number of unemployed and the weighted average rate of maximum weekly benefits. C/QPP revenues and expenditures are explicitly modelled.

Interest on the federal public debt is calculated using the stock of long-term bonds, CSBs and treasury bills and the related interest rates. Provincial-municipal interest payments are also calculated using outstanding bonds and interest rates.

EPF is modelled as a function of population times a synthetic EPF rate minus provincial income tax receipts. Equalization payments are expressed as a function of provincial direct and indirect tax revenue, and GNE. Canada Assistance Plan transfers are a function of provincial transfers to municipalities, the unemployment rate, and the ratio of the unemployment rate to the natural rate.

The various categories of government expenditures on goods and services are exogenous in real terms. A single deflator for government goods and services is applied to calculate nominal government spending.

### **3.10.10 MACE**

The government sector of MACE is highly aggregated and not broken down by level of government. One equation explains total direct taxes as a function of a personal tax rate times wage income plus transfers and other income. Another equation expresses corporate income taxes in the non-energy sector as a function of total revenue in the non-energy sector less wages and capital consumption allowances. Total indirect taxes are determined by non-energy absorption and exogenous (to the macro block) indirect energy taxes. Royalties and direct taxes on energy are also exogenous to the macro block.

Per capita government transfers to persons are a function of per capita income, the

unemployment rate and the absorption price. This includes interest on the public debt which is not modelled separately.

Government transfers to other levels of government are netted out.

Total government spending on goods and services is exogenous in real terms. The implicit price of government spending is a function of the price of absorption, the wage rate, and a time trend.

### **3.10.11 SAM**

There is also only one level of government in SAM. Revenue and expenditure categories are highly aggregated.

Personal income taxes in SAM are based on a reaction function approach. Personal tax rates are set so that the flow supply of government debt is sufficient to meet incremental demand. The deficit is thus not the residual in SAM as in other models.

Corporate income tax is tied to profits. Royalties are modelled as removing economic rents from the energy sector and equating returns to energy and non-energy investment at the margin. This permits the specification of a single private sector equity market for all capital.

Unemployment insurance contributions and C/QPP revenues and expenditures are not modelled separately. They are aggregated with personal direct taxes. Unemployment insurance benefits depend on the number of unemployed and benefits per unemployed member of the labour force which in turn is a function of the equilibrium wage.

Interest on the public debt, in both domestic and foreign currency, is explicitly represented by identities linking stocks outstanding to interest flows using endogenous average coupon rates. In the case of foreign-pay debt the influence of exchange rate changes is taken into consideration. The coupon rate equations update the average rate, taking into account net new issues and rollover of old debt in each period.

Government non-wage expenditure and transfers to persons are endogenous in nominal terms based on a target exogenous share of nominal domestic output with a cyclical response dependent on the gap between the actual and natural rate of unemployment. Government wage expenditure is the product of the government wage and government employment. The government wage is linked to the private sector wage. Government employment as a proportion of labour supply is a function of the gap between the actual and natural unemployment rate.

## **4 MODEL EVALUATION**

Model evaluation is necessarily a subjective exercise and depends on the ultimate application of the model as well as the quality of the underlying theory and structure. The standard which determines the criteria utilized in this paper is the contribution that the model and its various sectors can make to the development of a better QFS model given the objectives of the Economic Forecasting Division and the Department of Finance's Branch Model Development Committee. This section is thus designed to draw on the analysis of the sectors of the individual models to offer advice on possibly fruitful directions for research work on QFS. This is a necessarily impressionistic exercise. It does not attempt to answer the question which models have the best sectors. This of course depends on the purposes and use of the models which in many cases are quite different than those of QFS.

The criteria on which the evaluation is based are:

### **1. Theory**

- is the underlying theory current, and does it reflect economic developments?
- is the theory internally consistent?
- are the theoretical foundations sufficiently rigorous?

### **2. Structure**

- are demand and supply appropriate for the sector?
- are the linkages from the sector to the rest of the model well defined?

### **3. Practicality**

- are estimation techniques overly complex?
- are data quarterly, or could they be modified to quarterly frequency?
- is the estimation period current?
- are data easy to update and maintain?

### **4. Application to medium-term forecasting**

- are the steady state properties well defined?
- is the long-run consistent among variables within a sector?

### **5. Application to alternative forecast scenarios**

- does the model exhibit sensible dynamic properties?

## **6. Application to fiscal policy levers**

- are there appropriate fiscal policy levers?

The detailed sectoral descriptions above deal with the issues of theory and structure. This section pulls together from this the points of most relevance for QFS for each sector and model.

Before getting into the sector by sector and model by model discussion, there are some general points on practicality and documentation which pertain to the overall models as well as to sectors. The points are summarized by model.

CANDIDE 2.0 is now an out-of-date model. Its estimation period ended in 1974-1976, so its structure does not reflect recent history. A new version of CANDIDE is soon to be released and it warrants careful study. The documentation for CANDIDE is very good. This should facilitate analysis. It must be recognized, however, that CANDIDE is an annual model and that it would not be easy to incorporate any of its sectors into a quarterly forecasting model without substantial modifications. Also it is doubtful if the industrial disaggregation would be helpful in short- to medium-term forecasting. On the other hand, CANDIDE does possess a rich array of fiscal policy levers across most sectors that make it a suitable vehicle for fiscal policy analysis.

The TIM model is current with its estimation period ending from 1977 to 1980 with 1980 being the most frequent end-point. The documentation is good for those able to follow computer coding. TIM like CANDIDE is likely to be of less direct relevance for QFS given its annual data base and its industrial disaggregation.

RDX2 is now primarily of historical and theoretical interest. Its sample ended in the fourth quarter of 1972. There has since been a great deal of structural change in the economy, and theoretical development in the economic literature.

The quarterly models CHASE, DRI, FOCUS, MTFM and RDXF are most directly relevant for QFS research.

The CHASE model is current with its estimation period ending from 1978 to 1982 depending on the equation. One drawback with CHASE is that its structure is poorly documented. All that was available was a printout of the estimated equations and a brief but out-of-date description of the model's structure. The equations were not grouped by sector and thus it is not a straightforward task to ascertain how a given sector works or the links between the sectors.

The DRI model is current with its estimation period ending in the fourth quarter of 1982. The quality of the documentation is good. The only drawback is that the coefficients and estimation results are not provided.

The FOCUS model is starting to become out-of-date. Its estimation period ends from 1975 to 1977. Thus it is unlikely that many of its sectors would hold up under reestimation. A major model development effort is planned to begin this year, but it will be at least a year before it will bear fruit.

The MTFM model is more current with its estimation period ending from 1976 to 1981, with the fourth quarter of 1981 being the most frequent end-point. The end-point of the estimation period was selected to leave some post sample observations for testing. The model would be more current if after the testing was complete the model had been reestimated using the latest data then available. The documentation of MTFM is good.

RDXF is updated frequently. Substantial resources go into keeping it current and analyzing its properties. The end of the estimation period for the December, 1983 version studied varies from the fourth quarter of 1980 to the fourth quarter of 1982. The documentation is very good.

The MACE model is a very small annual model with only 50 variables and 25 stochastic equations. It was not designed for short- and medium-term forecasting. It lacks many fiscal policy levers because of its high degree of aggregation. On the other hand, its integration of a macro model and a detailed energy sector is unique and its theoretical basis is very rigorous. The model documentation is also good. While there is probably nothing that is directly transferable from MACE to QFS, there is much to be learned from a study of its structure.

SAM is another small (129 variables and 22 stochastic equations) annual model. It too was not designed for short- to medium-term forecasting, but is intended for medium- to long-term simulation analysis. The particular focus of SAM is the steady state and the process of adjustment to that state. The structure of SAM is thus necessarily theoretically very rigorous. Many constraints had to be imposed in estimation to ensure consistency. The emphasis was not on short- to medium-term tracking, but on longer-run properties. This is a perfectly valid approach given the objectives of SAM. It made possible a major contribution to the understanding of long-run dynamics. On the other hand, it is not the most suitable for the development of a quarterly forecasting model in that much necessary detail such as the housing sector had to be sacrificed. The equation structures and estimation techniques are in many cases too complex to be applied to larger systems of equations such as are typically found in quarterly forecasting models. The steady state is not something that dominates short- to medium-term economic trends, although its influence on shorter-term dynamic paths is probably understated in most quarterly models. The documentation for SAM is very good.

#### **4.1 Financial Sector**

The CANDIDE financial sector is relatively complete including a government budget constraint. Its theoretical basis is sound. It would be difficult to transform it to quarterly because of the



importance of lags in the financial sector.

The TIM financial sector is fairly basic.

The RDX2 financial sector was theoretically the most rigorous of the quarterly models with its constrained estimation of the general public's portfolio. The money demand function in the model depended on the size of the financial portfolio. This specification has fallen from favour and the new money demand equations are based only on income and interest rates. Also there has been much change in the structure of the financial sector since RDX2 was estimated.

The CHASE financial sector is very extensive. The money demand function has a small response to interest rate changes. Long-term capital flows are not modelled.

The DRI model has a relatively complete financial sector, which is well based theoretically. An important weakness is the absence of a government budget constraint. The money demand function also has an implausibly low elasticity on prices and on the deposit rate.

The FOCUS financial sector is not very disaggregated, lacking complete balance sheets for banks and other financial institutions and a government budget constraint. Interesting features of the financial sector are the number of switches to choose the monetary regime and the consistent use of synthetic expectations. A problem in the financial sector is the high degree of sensitivity of capital flows to interest rate changes. This causes peculiar over-all model results when interest rates are shocked.

The MTFM financial sector is complete including a budget constraint and capital flows.

The documentation provided on the RDXF financial sector had key equations missing so it is impossible to evaluate the sector fully. Nevertheless, there is an extensive menu of interest rates and financial assets and liabilities modelled.

MACE has a very small financial sector which only includes the demand for high powered money and business assets.

SAM only has a small number of assets, but its financial sector is still very complex, particularly in how it interacts with the real sector and in its disequilibrium adjustments.

## **4.2 Wealth, Permanent Income**

Wealth plays no key role in the real sectors of any of the large annual or quarterly models. It is only in the small models MACE and SAM that wealth is really important. This is probably

indicative of the great difficulty in determining wealth and in obtaining wealth data on a timely enough basis that it can be of assistance in forecasting.

Permanent income is not explicitly defined in CANDIDE, TIM, CHASE, or RDXF. It is calculated in DRI, FOCUS, MTFM, QFS, and RDX2. DRI, FOCUS, and QFS definitions allow for trend growth; MTFM and RDX2 do not. None of the models with trend growth allow for a slowdown corresponding to the overall slowdown in aggregate productivity after 1973. This would appear to be necessary to ensure consistency, and research in this area should be considered for QFS.

The QFS weight on current income at .22 was the highest of all the models. This could perhaps help to explain why the MPC in QFS is so much lower than in other models.

CHASE, RDXF, and MTFM exclude some portion of the inflation premium on interest income from adjusted disposable income. This is worth considering for QFS. MTFM also adjusts interest on the public debt to reflect the extent to which it is perceived to be associated with higher future taxes. This too is an interesting approach. More importantly, it raises some questions about the long-run effectiveness of fiscal policy which might usefully be examined in more detail. It would be useful to perform statistical tests of the significance of some of these new permanent income concepts and related parameters to see if their inclusion would improve the consumption sector of QFS.

### **4.3 Wages and Prices**

The CANDIDE wage and price sector is grounded in basic economic theory and its dynamic properties are well tested and documented. The wage sector incorporates the prime age male rate as the key labour market tightness variable. There is no productivity growth term, however. Prices are based consistently on the input/output framework. This would not be possible in a quarterly model. The use of the actual rather than normal unit labour costs is debatable. Another problem in the wage-price sector is the inclusion of money growth in expected inflation. This introduces a significant source of non-homogeneity in the wage price block.

The TIM wage sector uses the inverse of the unemployment rate as the labour market tightness variable. This is inappropriate if the natural rate has changes over the period. There is no allowance for variations in productivity in the wage equation. The price sector also depends on the full blown input/output framework which is not a viable option for a quarterly model. TIM also uses actual unit labour costs in price equations.

The theory behind the RDX2 wage sector was developed before the extended Phillips curve gained widespread acceptance and thus is no longer consistent with current theory. The price sector, however, is still state of the art. The use of capacity utilization variables and inventory

disequilibrium variables consistently derived from the supply side are a particularly noteworthy feature of the sector.

The key wage equation in the CHASE model is, essentially, considered on an *ad hoc* empirical basis. The interrelationships of the various wage series, on the other hand, resemble Phillips curves. The price block of CHASE is standard.

The key wage equation in the DRI model for average hourly earnings in manufacturing is specified on a fourth-over-fourth basis, and does not allow for variations in trend productivity growth. The price sector is interesting in the way it utilizes the input/output approach. This could have some applicability to QFS if industrial detail were thought to be desirable.

The FOCUS model has a small but theoretically rich wage-price sector. The wage equation incorporates consistent price expectations, catch up, and the impact of product prices relative to sales. It lacks unfortunately an explicit productivity term. The price sector has one key price which feeds into all of the other prices. This approach is an alternative to the specific modelling of disaggregated prices and should be considered, but not necessarily adopted, for QFS. The flexible price option of FOCUS is another feature which is potentially useful for certain types of policy simulations.

The MTFM wage sector is not based on extended-Phillips curve theory, but instead is a real wage formulation. It uses the total unemployment rate as its labour market tightness variable. The price sector of MTFM is based on a stage of processing model, and is worth considering as a possible approach for QFS.

The RDXF wage equation looks very good. It is based on current theory omitting no important variables except perhaps catch up, and appears to incorporate reasonable lags. The RDXF price sector also appears to be very well constructed.

The MACE wage and price sector is theoretically rigorous, but simple and straightforward. It incorporates the most important explanatory variables, and is tied in nicely with the supply side.

The SAM wage and price sector is also theoretically rigorous, but very complex. The complexities are necessitated by the need to impose steady state properties which is the main objective of SAM. Such complex specifications are neither necessary nor desirable in a quarterly forecasting model.

#### **4.4 Aggregate Supply**

The CANDIDE supply side is based on input/output. A fully articulated input/output framework

would not be a possibility for a quarterly forecasting model. There are no production functions in CANDIDE, therefore, factor demands are not consistently derived. On the other hand, the investment sector has a wide range of fiscal policy instruments by industry that make the model useful for fiscal policy analysis. A drawback of the investment sector which is remedied in the upcoming version of CANDIDE is the use of nominal interest rates in the user cost.

The TIM supply side is also based on input/output and is not readily adaptable to quarterly forecasting. It also has no explicitly formulated production functions and interrelated factor demands. A nominal interest rate is utilized in the user cost, which does not incorporate tax factors.

RDX2 remains the only member of the RDX family to have a fully consistent framework of interrelated factor demands. It still serves as a good guide as to what an integrated supply side should look like.

The CHASE model has a central production function, but its factor demands are not consistently estimated. Investment is based on the accelerator and cash flow and allows no role for interest rates or tax factors. Employment is based on dividing nominal output by the wage rate. This involves a certain circularity because the key wage equation is also based on a share specification.

The DRI model also has a central production function which is not utilized to estimate interrelated factor demands. The user cost in the investment equations utilize a nominal interest rate, which is inappropriate. On the positive side the important tax parameters are present.

The FOCUS model is based on an underlying production function which, while slightly idiosyncratic, is theoretically rigorous. The determination of factor demands from this production function is consistent. The FOCUS supply side is sufficiently different and interesting that it merits careful examination by those concerned with developing the QFS supply side.

The MTFM model supply side has no production function and here factor demands are not estimated consistently within an interrelated framework. It is of interest primarily for what it shows about how industrial disaggregation can be introduced in a quarterly model in an integrated fashion (as opposed to CHASE and DRI where the industrial disaggregation is recursive).

RDXF started out with a central production function and consistently related factor demand equations. Somehow however, the consistency was lost in the subsequent revisions to the model. The Bank's supply side project when implemented will presumably remedy these deficiencies.

The MACE model has a production function and a consistent structure of interrelated factor

demands. Its production function incorporating energy is well developed and theoretically rigorous. Analysis using it has provided some very interesting and thought provoking observations on the role of energy and other factors in the productivity slowdown. It certainly warrants careful study by anyone concerned with developing a supply side for a macro model. Its modelling of interrelated factor demands and the residual role of inventories also has some valuable lessons to offer those who will be involved in the supply side of QFS. The investment sector includes housing and is probably too aggregated to be of much use to those primarily interested in quarterly forecasting. It also lacks the necessary instruments for fiscal policy analysis.

The supply side of SAM is very innovative. Its integration of steady state and disequilibrium dynamics contributes a real step forward in model-building. The demand for factors of production

is consistent with the production function. A drawback of SAM from the point of view of developing a quarterly forecasting model is that the investment and employment equations are probably too complicated to be incorporated in a quarterly forecasting model. Also the aggregation of all investment including housing in one equation renders the sector unsuitable for quarterly forecasting.

#### **4.5 Income Distribution**

The key issue in the income distribution sector is the treatment of corporate profits. In theory, it does not matter whether a model has an explicit equation for corporate profits or if corporate profits are residually determined. Both approaches have been followed by some of the models studied. CANDIDE, TIM, CHASE, MTFM, and RDX2 all treat profits as a residual. DRI, FOCUS, QFS, and RDXF have a profits equation. There is a certain appeal to treating profits as a residual since in some sense they are the residual after other costs of production have been covered. On the other hand, there are also advantages from a forecasting point of view in explicitly modelling the behaviour of profits. If profits are modelled, however, there is the problem of what to do with the discrepancy between the income and expenditure side of the accounts. DRI and FOCUS, following their predecessor QFM, both include the discrepancy in disposable income by counting it as unincorporated business income. QFS and RDXF in contrast spread the discrepancy. A problem with either of these approaches is that it introduces an additional dynamic element on the income side that can cause difficulties in simulation.

The two large annual models and the quarterly models all have fairly detailed income sectors. When information on the best specification for a particular income side component is required, they should be consulted. The income distribution sectors of the two small annual models is rudimentary and offers no guidance on income sector detail.

## **4.6 Housing**

Housing is an important sector in models used for short-term forecasting and policy analysis. While the housing sector itself is small, it accounts for a disproportionate share of cyclical variations in the economy. This stems in large part from its sensitivity to fiscal and particularly monetary policy. It has thus been the focus of much concern and the target of many specific policy initiatives. This makes it extremely difficult to model, but all the more important to understand.

The two main approaches to modelling housing are either to link housing starts to mortgage approvals or to model housing starts directly. RDX2 was the original model to follow the mortgage approvals path. The advantage of this approach was seen to be that it ensured consistency between the financial and real sides of the housing market and was a convenient way to incorporate financial factors. CANDIDE and FOCUS have followed RDX2 in modelling housing starts based on mortgage approvals.

The second approach of modelling housing starts directly has been pursued by TIM, CHASE, DRI, MTFM, QFS, and RDXF. TIM and DRI housing sectors incorporate only demand elements, whereas CHASE, MTFM, QFS, and RDXF have a role for supply factors as well as demand. The CHASE, QFS, and RDXF housing sectors can not be viewed as independent. The CHASE and QFS sectors were based on that of RDXF. The recent version of CHASE puts more emphasis on demand factors in the housing starts equation.

Housing is a problem sector. The mortgage approvals approach of RDX2, CANDIDE, and FOCUS is unsatisfactory because it gets the direction of causality in the housing sector wrong. The pure demand real sector approach of TIM and DRI is incomplete, although DRI's housing burden concept is of interest. An examination of the housing sectors in the other models is also not very encouraging. It may well be that the only viable option is to start from scratch and build a structural model of the housing market that will reflect the behaviour of all agents and clearly specify demand and supply in all sub-markets.

## **4.7 Consumption and the Savings Rate**

An important issue in the consumption sector is the appropriate degree of disaggregation for a quarterly forecasting model. The large annual models have very large consumption sectors. CANDIDE has 48 consumption categories and TIM 80. This is mainly required to provide an expenditure breakdown at the level of detail necessary for the input/output decomposition. It is certainly not needed to improve forecasting or simulation performance, unless industrial outlooks and issues are involved.

Consumption sectors in the quarterly models have tended to grow in size since RDX2. The number of categories falling from the largest to the smallest are DRI (11 categories), RDXF and CHASE (9 categories), QFS and MTFM (8 categories), FOCUS (6 categories), and RDX2 (5 categories).

The consumption sectors of MACE and SAM only include a single equation. This is not enough for a quarterly forecasting model where, particularly in the short-run, variations in a particular consumption category can have a significant impact on overall consumption.

The savings rate approach to modelling consumption utilized in CANDIDE is of interest even though it has been dropped in the upcoming version of CANDIDE. It facilitates the analysis of some questions by isolating the impact of changes in discretionary and non-discretionary savings on consumption. It might be of particular utility in analyzing policies such as pension reform and changes in registered savings plans. There also could be certain forecasting advantages in focussing on the factors that cause changes in the savings rate. These of course would have to be weighed against the disadvantages of the approach. For instance, it could well be that variations in demand for particular categories of spending, most notably durables, that cause a certain portion of the fluctuations in savings rates rather than vice versa.

The TIM model has a concept called discretionary income which might be of interest. It is defined to be disposable income minus spending on necessities, and it is used to explain spending on goods other than necessities.

The DRI and FOCUS models have automobile demand equations incorporating the user cost of owning and operating a car. While QFS also utilizes user cost, it does not incorporate the cost of operating a car. It might be useful to consider the adoption of a broader concept of user cost in QFS.

The SAM consumption equation embodies complicated expressions to adjust valuations for the divergence of wealth and prices from their steady state values. This type of specification would be overly complicated and unnecessary for a quarterly forecasting model.

Another issue is the impact of interest rate changes on consumer expenditures. A problem in all of the models except MACE and SAM is that interest rate increases raise the non-wage income component of disposable income and thus have the partial impact of augmenting consumption. This perverse partial impact is only offset if the direct impact of interest rates on consumer expenditures, particularly durables, is sufficiently strong. Some of the quarterly models such as RDXF and CHASE exclude the inflation premium on interest income from disposable income. This prevents increases in nominal interest rates arising from inflation from raising consumer expenditures, but does not eliminate the partial perverse impact of real interest rate increases. The solution to the problem of the perverse partial impact of monetary policy on

consumer expenditures probably lies in the incorporation of interest rate induced changes in the value of financial wealth in consumer expenditure equations.

#### **4.8 Labour Supply**

CANDIDE 2.0 and TIM both contain detailed demographic blocks and disaggregated participation rate equations. This may be a real advantage in modelling labour supply. The CANDIDE part rate equations are plausible enough by themselves, but there are reasons to be skeptical about the strength of the labour supply response to tax changes, which in simulation can almost counteract the impact of labour demand on the unemployment rate.

RDX2 also only had a single participation rate equation. It included the capacity utilization rate, population in secondary school, and net immigration as explanatory variables.

The CHASE model has six age-sex groups. Its part rate equations, which depend on a trend and the percentage change in employment, appear to be misspecified. The trend is the only influence captured substantially by the equations.

The DRI model does not explicitly model labour supply. Instead it sums employment and unemployment. The key relationship is an Okun's law equation relating the gap between the actual and natural unemployment rate to capacity utilization and the ratio of the prime age male labour force to the total working population. Such an equation does not directly reflect many of the important factors behind decisions to participate in the labour force affecting various age and sex groups, and is a questionable basis for determining labour supply.

The FOCUS model has an age-sex breakdown and structural participation rate equations incorporating age and sex specific factors as well as general trends and employment rates. The only problem with the sector is that it is getting out of date and may not reflect recent movements in part rates that may reflect structural changes.

MTFM also has an age-sex breakdown and structural participation equations that appear reasonable.

RDXF only has a single participation rate. This may not be adequate to capture the complex age and sex specific factors determining the participation rate. The lag structure of the equation has also grown increasingly complex since the original version of RDXF. This could reflect an attempt to pick up with lags what are really shifts in fundamental determinants.

The MACE part rate equation is interesting in that it suggests that the AIB raised part rates and that an increase in real wages lowers the part rate. This latter result is opposite from that found in CANDIDE.

In SAM labour supply is based on the same intertemporal utility maximization decision as



consumption. As a result, the specification is very complicated involving the ratio of per capita real wealth to the returns from labour market participation and adjustments for the difference between steady state and actual valuations.

Such a specification is unlikely to track in the short- to medium-term because of its lack of labour market slack variables and trends related to demographics. It would not be suitable for a quarterly forecasting model.

#### **4.9 Trade**

An important issue in the trade sector is the extent of disaggregation that is desirable. The large annual models, CANDIDE and TIM, are able to take advantage of the extensive detail available in the annual data. CANDIDE has 208 trade volume and value equations and 89 trade price equations and TIM has 730 trade equations and 234 trade price (including identities). It would not be possible for a quarterly model to have such a high level of disaggregation.

The quarterly models range in size from MTFM with 117 trade equations and 131 trade price equations and QFS with 81 trade equations and 86 trade price equations to FOCUS with 37 trade equations and 19 trade price equations. In the middle fall CHASE (63 trade and 32 trade price equations), DRI (49 trade and 33 trade price equations), and RDXF (44 trade and 28 trade price equations). Somewhere in this range is probably about the right size trade sector for a quarterly forecasting model.

The two small annual models have very small trade sectors. MACE has 6 trade equations and 2 trade price equations and SAM 2 trade equations and 4 trade price equations. This would clearly not be enough for a quarterly forecasting model because quite often the short-term growth of exports and imports are associated with particular developments affecting certain categories of trade flows.

A minimum level of disaggregation for forecasting the trade sector would include separate treatment of exports of wheat, automobiles, forest products, minerals, and energy and of imports of automobiles, energy, and interest and dividends. Further disaggregation could be helpful provided sufficient attention were paid to getting the detail right.

Among the quarterly models the approach to modelling trade flows is very similar. This is particularly the case for import equations where the general specification is as a function of domestic activity, relative prices, and capacity utilization. For export equations there are some differences in the extent to which supply variables play a role, but there is general agreement on the importance of trade weighted foreign activity variables and some measure of export prices relative to foreign prices. Concerning import prices, all of the models exhibit price taking behaviour. But for export prices there is disagreement. Export prices in RDXF and CHASE primarily reflect price taking behaviour, whereas in DRI and QFS export prices are set in Canada. FOCUS, MTFM, and RDX2 display both price setting and taking behaviour to varying degrees. In this case the middle road would seem to be the most reasonable course. For some categories of

exports, including particularly, primary products and raw materials, price-taking behaviour is probably dominant; for others such as manufacturing where there is more product differentiation, price-making behaviour may be more characteristic.

The SAM export equation is quite different from the other export equations. In the long-run exports are determined by supply. This assumption might be useful for ensuring that the trade balance does not become a source of instability in longer-run simulations but it does not give enough emphasis to foreign demand for short- to medium-run forecasting.

The exchange rate equations in those models that have them are essentially renormalized short-term capital flow equations. Since this theoretical approach to the exchange rate suggests that the appropriate form for such an equation is as an adjustment of desired to actual stock of outstanding short-term liabilities, a well specified equation should have the lagged stock of outstanding short-term liabilities as an explanatory variable. This was the case in RDX2 and is the case in DRI and CHASE, which followed RDX2.

Other models, particularly, RDXF and QFS, take a purchasing-parity view of longer-term exchange rate determination, although there are capital flow influences in the short-term. One question in such models is what index should be used to measure PPP.

FOCUS takes another approach to determining the exchange rate. It is set as the price which equilibrates the balance of payments. This is possible because of the high degree of sensitivity of capital flows to exchange rate variations. The FOCUS approach certainly represents an interesting alternative. However, the high degree of sensitivity of capital flows does cause FOCUS to respond too strongly to changes in interest rates. So it is not clear that a market clearing exchange rate is a viable approach for a quarterly model.

MACE has a large array of options for exchange rate determination implemented through simulation rules. This is useful for policy analysis.

SAM determines the exchange rate as clearing the market for foreign assets. This is a consistent and theoretically rigorous approach which integrates well with its treatment of other financial assets.

#### **4.10 Government**

There is relatively little that the Department of Finance can learn from the government sectors of the macro-models. As a general rule the models do not incorporate the degree of detail required, nor is sufficient attention always paid to institutional details and the specification of individual equations. This is understandable given that the primary objective of the models is to explain aggregate economic activity and not to prepare the government's fiscal plan.

A feature of two of the models, RDX2 and CANDIDE, that may be of some interest is their personal income tax models. An up-to-date version of such a model could be of some use in producing medium- and long-term fiscal forecasts.

## **5 SOME OBSERVATIONS ON CANADIAN MACROECONOMIC MODELLING AND PROMISING AREAS FOR FUTURE RESEARCH**

Technology has opened new frontiers. Supercomputers a thousand times more powerful than yesterday's mainframes now exist. The availability of raw computing power no longer is a constraint as it was in early modelling efforts. The next generation of 32 bit desk-top microcomputers will be able to run the largest of today's models. Supermodels 10 to 20 times larger can be handled by mainframes.

Important fundamental constraints on macroeconomic modelling still exist. The greatest constraint of all is probably the inability of economists to capture mathematically in models the workings of hugely complex modern economies. Nevertheless, great strides have been accomplished. As economic theory develops and is better applied, and as the quality and quantity of data on the economy improves, further progress will be made in macroeconomic modelling.

A less fundamental constraint is our ability to understand and manage a model. In any modelling project there must be at least one individual who fully understands all of the model's parts and how they fit together. The larger and more complex the model, the more difficult the task. The human brain becomes the ultimate constraint on the size and complexity of a model.

A dissatisfaction with the way models have been put together and the lack of understanding of long-run properties have led researchers at the Bank of Canada to put together SAM. The small size of the model has enabled its builders to utilize much more complicated specifications and have a greater degree of simultaneity. The imposition of a priori constraints and the interrelated nature of many sectors have necessitated the use of more sophisticated estimation techniques.

The MACE model is also small, but was not made small to allow its builders to increase its complexity. Instead its builders believed that existing models were getting too large and fragmented and wanted to develop a model that was small enough to guarantee both timeliness and economic coherence. The MACE model reflects this approach. It is a small model with a simple yet theoretically rigorous structure suitable for general macroeconomic analysis.

Modularity is another way of relaxing the human constraint. If sectors or sub-models can be segmented in reasonable ways and their interactions can be clearly specified, then it is possible to build models in a modular fashion. This enables larger models to be built.

Satellite models offer a closely related approach that holds some promise of overcoming the comprehensibility and complexity problem. Core models can be kept as simple as possible. Satellite models can be plugged in replacing a particular sector or sectors for specific applications, provided the appropriate interfaces with the core model exist.

For many of the most interesting applications, the interfaces will have to allow for two-way feedbacks. This renders the development of general core models more difficult.

The development of increasingly sophisticated software for model-building and management will help to relax the human constraint. TROLL, TSP and other advanced scientific programming packages are not the last word in econometric and simulation software and improvements can be made that will lessen the burden of updating data bases, estimating equations, assembling equations into sectors, simulating sectors and the complete model, analyzing the model's structure and dynamics, performing tracking simulations, and preparing documentation. These are the mundane tasks that occupy most of the model-builder's time. If they become less burdensome, the model builders will both have more time and be better informed in making the key choices that are at the heart of model-building.

One particular futuristic possibility that was raised in discussions with modelling groups was the use of artificial intelligence front-ends to facilitate the building, management and use of large models.

The the development of software for model-building and management since RDX2 has been in many respects remarkable. The old RDX2 simulator and estimation and database packages were relatively cumbersome in relation to the new generation of interactive programs such as TROLL and TSP.

The degree of progress in the development of the models themselves has been somewhat less impressive. In such areas as the supply side and interrelated factor demands the present quarterly models actually represent a step backwards. In other areas models have continued to develop in line with the new theory and events. Examples include the adoption of the extended Phillips curve as the accepted wage specification, the recognition of the productivity slowdown, the incorporation of energy, and the modification of financial sectors to reflect monetary targeting. Quarterly models have also introduced industrial disaggregation. This has been done by private sector forecasting agencies to meet the demands of their clients.

It is very difficult to foresee the future course of Canadian macroeconomic modelling. It will of course be influenced by developments in theory, data, events and technology, not necessarily in that order. Efforts will be made to remedy obvious deficiencies in existing models.

For the large annual models based on input/output the availability of input/output time series opens up new possibilities. For small annual models, the spread of microcomputers is likely to make them more widely available. The FAIR model in the U.S. is already on the market. A group

at Laval has mounted MACE on a microcomputer. The Bank is considering the possibility of providing SAM on a floppy disk. This will give many more people, particularly students and professors in universities, an opportunity to become familiar with macro-models and how they work. It could help to train a new generation of modellers. It may spawn interest in developing satellite or special purpose models.

This too can be done on a microcomputer.

There are an number of promising areas for future research that can be identified. The specification of the supply side of macro-models and its consistent integration into the overall model framework should remain a priority for model-builders. The task will be particularly difficult for the models with industrial disaggregation.

The treatment and role of expectations is another area requiring future research. This would be much aided if some attention were to be paid to the development of price and output expectation series. Surveys offer one promising approach for obtaining data on expectations. However, it will be quite a while before consistent time series are available and it will be even longer if the process does not start soon.

In keeping with the research on the supply side and expectations, it will be important to focus much more on the overall dynamic properties of macro-models and ensuring consistency among sectors. This will require a rigorous program of complete and partial simulation analysis.

Various recent developments will no doubt be fruitful topics for model related research. The sharp and substantial rise in the level of real interest rates since 1980 has been out of the bounds of earlier historical experience and has no doubt had widespread impact on the economy. This offers an opportunity for researchers to learn more about the links between the financial and real sectors.

The high and volatile savings rate is another issue that should be explored. Important questions are the extent to which it has been influenced by inflation and by nominal and real interest rates. Also it is important to understand the significance of various institutional factors. A two way attack on the savings rate would appear to be most promising - directly as in CANDIDE 2.0 and indirectly through the estimation of disaggregated consumption functions. In the estimation of consumption functions it would be useful to explore the role of wealth. This is a variable that is very important in theory, but that has not been much utilized in larger macro-models because of the difficulty of obtaining high quality and timely data.

Another important issue that can be addressed through macro-models is crowding-out. The influence of deficits and debt on asset demand, interest rates and the exchange rate can be analyzed empirically in macroeconomic models. It would also be useful to examine the possible international transmission of crowding-out from the United States.

At a lower level of generality, the housing sector of most models is generally unsatisfactory. There might be some payoff to attempting a more detailed modelling of the various components of the housing market and of the behaviour of the main decisionmakers involved.

## **APPENDIX 1**

### **MODEL SUMMARIES**

#### **CANDIDE 2.0**

Date of Model: October, 1976

Use of Model: medium- and long-term forecasting; alternative forecast scenarios; fiscal and monetary policy analysis

Number of Equations: 2361 total; 715 stochastic

Frequency of Data: Annual

Computer System: CANDIDE simulator adapted from Wharton

End of Estimation Period: varies from 1974 to 1976

#### **Characterization of Sectors**

Consumption: personal saving divided into discretionary and contractual; nominal consumption is determined as a residual through personal income-saving-consumption identity; individual consumption categories are function of total consumption and relative prices

#### **Investment in Residential**

Construction: single and multiple dwelling starts are by demand factors, such as mortgage approvals, demographic factors, relative advantage of renting versus owning, and by supply factors such as lagged change in vacancies and government policy actions.

Investment: derived from neoclassical investment theory assuming profit maximization

Government Expenditure: federal/provincial split; behavioural model

Exports: demand; price taker

Imports: demand; price taker

Prices: cost add-up based on IO disaggregation and incorporating capacity variables

Price Expectations: influenced by past rates of change in CPI and M1

Technology: Implicitly Cobb-Douglas and/or CES by industry

Factors of Production: capital M&E; capital non-residential; man-hours;disaggregated by industry

Wages: extended Phillips curve by industry with the unemployment rate of prime age males as labour market tightness variable; in some equations U.S. wage rates and industry specific productivity is also included.

Exchange Rate: expected exchange rate is function of ratio of expected Canadian prices to U.S. and level of reserves; actual exchange rate is function of expected exchange rate, short-term capital flows, and percentage change in 3 month commercial paper rate

## **TIM**

Date of Model: March 7, 1984

Use of Model: medium- and long-term forecasting; alternative forecast scenarios; fiscal and monetary policy analysis

Number of Equations: 3914 total; 1206 stochastic

Frequency of Data: Annual

Computer System: SIMSYS, proprietary system of Informetrica; timesharing

End of Estimation Period: varies from 1977 to 1980; 1980 most common

### **Characterization of Sectors**

Consumption: Houthakker-Taylor specification

Investment in Residential

Construction: primarily demand-driven stock adjustment model

Investment: disaggregated by industry; specifications generally

based on Jorgenson neoclassical model, but also on cash flow and flexible accelerator models

Government Expenditure: federal/provincial split; many categories of spending are endogenous

Exports: demand; primarily price taker but for some items prices setter

Imports: demand; price taker

Prices: cost add-up based on input/output disaggregation including capital costs

Price Expectations: adaptive

Technology: Cobb-Douglas by industry with some exceptions

Factors of Production: capital M&E; Capital non-res; man-hours; disaggregated by industry

Wages: Modified Phillips curves by industry tied to manufacturing but with role for industry specific labour market conditions

Exchange Rate: simulation rule relating changes in the exchange rate to changes in real current account balance

## **RDX2**

Date of Model: 1976

Use of Model: simulation and policy analysis

Number of Equations: 325 total; 166 stochastic

Frequency of Data: Quarterly

Computer System: Simulator and Massager estimation package written in FORTRAN

End of Estimation Period: 1972Q4

### **Characterization of Sectors**

Consumption: permanent income with separate treatment of wage and non-wage income and including wealth

Investment in Residential

Construction: availability of mortgage funds determines demand



Investment: forward-looking flexible accelerator vintage  
neoclassical model with cash flow features

Government

Expenditure: federal/provincial split; behavioural model

Exports: demand; both domestic costs and foreign prices determine  
export prices

Imports: demand; price taker

Prices: cost mark-up

Price Expectations: adaptive

Technology: Cobb-Douglas

Factors of Production: capital M&E; capital non-res; man-hours

Wages: can be interpreted either in terms of factor share  
equation or extended Phillips curve

Exchange Rate: separate models for fixed and floating exchange rate  
regimes; floating exchange rate equation is renorm-  
ilization of net private demand for foreign exchange;  
explanatory variables are the basic balance, interest  
differential, the ratio of foreign to domestic real  
wages, and net short-term liabilities

## **CHASE**

Date of Model: April 30, 1984

Use of Model: short-, medium-, and long-term forecasting; alterna-  
tive forecast scenarios, fiscal and monetary analysis

Number of Equations: 924 total; 319 stochastic

Frequency of Data: Quarterly

Computer System: EXSIM, WORLDSIM; Chase Econometrics System

End of Estimation Period: varies from 78 to 82

### **Characterization of Sectors**

Consumption: permanent discretionary income defined as lag on  
disposable income minus personal debt payments and the  
inflation premium on net personal wealth

## Investment in Residential

Construction: housing starts per capita are a function of sales price relative to construction costs reflecting supply and real effective purchasing power per capita and the mortgage rate reflecting demand factors: the housing price relative to the consumer price index is an inverted housing demand equation and is a function of real discretionary income per capita, the mortgage rate, and the stock of existing housing per capita

Investment: accelerator with financing constraint

Government Expenditure: federal/provincial split; exogenous real

Exports: demand; primarily price taker

Imports: demand; price taker

Prices: cost mark-up

Price Expectations: adaptive

Technology: Cobb-Douglas for private business product

Factors of Production: capital M&E; capital non-res; man-hours

Wages: percentage change in wage bill plus supplementary labour income in industrial composite depends on percentage change in private business product and the change in the gap between the actual and natural rate of unemployment

Exchange Rate: function of short-term interest rate differentials, the trade balance, capital flows, and a purchasing power parity term

## **DRI**

Date of Model: 1983B Summer 1983

Use of Model: short- and medium-term forecasting; alternative forecast scenarios; fiscal and monetary policy analysis

Number of Equations: 674 total; 226 stochastic

Frequency of Data: Quarterly

Computer System: EPS, proprietary system of DRI; timesharing  
End of Estimation Period: dependent on data availability;  
1982Q4 for most equations

### **Characterization of Sectors**

Consumption: permanent income

Investment in Residential

Construction: key equation for housing starts primarily demand driven

Investment: cash flow augmented neoclassical stock-adjustment model embodying a replacement investment hypothesis

Government Expenditure: federal/provincial; current federal are exogenous in either real or or nominal terms; current provincial can be either endogenous or exogenous, if exogenous, can be nominal or real; federal and provincial investment are exogenous in real terms

Exports: demand; primarily price setter

Imports: demand; price taker

Prices: cost mark-up using input/output data to relate industry prices to final demand prices; industry prices are explained in a stage-of-processing price model

Price Expectations: adaptive

Technology: Cobb-Douglas in aggregate; stage-of-processing by industry

Factors of Production: capital M&E; capital non-res; labour; energy

Wages: extended Phillips curve

Exchange Rate: eclectic with interest rate differentials, basic balance, portfolio considerations; and expectations

### **FOCUS**

Date of Model: February 13, 1982

Use of Model: policy and simulation analysis over the short-,  
medium-, and long-term  
Number of Equations: 344 total; 150 stochastic  
Frequency of Data: Quarterly  
Computer System: FORTRAN simulation and estimation package at  
University of Toronto Computer Center.  
End of Estimation Period: varies; mostly from 75 to 77

### **Characterization of Sectors**

Consumption: permanent income

Investment in Residential

Construction: availability of mortgage funds determines demand

Investment: non-residential construction function of difference between  
after-tax real rate of return on investment and real  
interest rate; machinery and equipment is based on  
modified Jorgenson model; cash flow affects rate of  
adjustment

Government Expenditure: federal/provincial split; real exogenous

Exports: demand; some price taker; others price maker

Imports: demand; price taker

Prices: Two Options: market clearing; and mark-up

Price Expectations: synthetic expectations based on regression of  
inflation on variables considered important  
by market participants; used consistently  
throughout model

Technology: Cobb-Douglas for private sector made up of  
N identical plants

Factors of Production: capital M&E; production labour

Wages: extended Phillips curve

Exchange Rate: determined as market clearing price for disaggregated capital flows/balance of payments sector

### MTFM

Date of Model: October 24, 1983

Use of Model: short- and medium-term forecasting; alternative forecast scenarios; fiscal and monetary policy analysis

Number of Equations: 893 total; 334 stochastic

Frequency of Data: Quarterly

Computer System: TROLL; timesharing available through Conference Board

End of Estimation Period: varies from 1976 to 1981; mostly 1981

#### Characterization of Sectors

Consumption: eclectic including permanent income, relative prices, expected real rate of interest and consumer confidence

Investment in Residential

Construction: equations for single and multiple units incorporate demand elements and government housing assistance and a supply element, the profitability of housing starts

Investment: Jorgenson neoclassical investment model for a number of industrial categories

Government Expenditure: federal/provincial split; real exogenous

Exports: supply and demand; price taker for some commodities, price setter for others

Imports: demand; price taker

Prices: cost mark-up incorporating stages of processing

Price Expectations: adaptive

Technology: no explicit production function; implicit production functions in employment demand and investment equations are Cobb-Douglas

Factors of Production: in industry employment equations have M&E

and non-res

Wages: reduced form equations by industry incorporating both labour supply and demand factors

Exchange Rate: inverted short-term capital flow equation; determined by basic balance, and change in interest rate differential; does not include a lagged stock adjustment term for short-term capital

### QFS

Date of Model: December, 1983

Use of Model: short- and medium-term forecasting; alternative forecast scenarios; fiscal and monetary policy analysis

Number of Equations: 688 total; 196 stochastic

Frequency of Data: Quarterly

Computer System: TROLL

End of Estimation Period: 1981Q4

#### **Characterization of Sectors**

Consumption: permanent income with relative prices and user cost for automobiles

Investment in Residential

Construction: housing starts determine supply and MLS price determines demand

Investment: Jorgenson neoclassical model

Government Expenditure: federal/provincial; real exogenous

Exports: mostly demand; primarily price setter

Imports: demand; price taker

Prices: cost mark-up with inventory disequilibrium term

Price Expectations: adaptive

Technology: Cobb-Douglas for real gross private business product;

various other specifications utilized

Factors of Production: capital M&E; capital non-res; labour; energy

Wages: extended Phillips curve

Exchange Rate: purchasing power parity in the long-run; interest rate differentials and movements in the basic balance are significant in the short-run

### **RDXF**

Date of Model: December 1983

Use of Model: short- and medium-term forecasting; alternative forecast scenarios; fiscal and monetary policy analysis

Number of Equations: 431 total; 192 stochastic

Frequency of Data: Quarterly

Computer System: TSP

End of Estimation Period: varies; mostly 1980Q4 to 1982Q4

#### **Characterization of Sectors**

Consumption: permanent income excluding the inflation premium with relative prices and financial variables

Investment in Residential

Construction: housing starts determine supply and MLS price determines demand

Investment: neoclassical model

Government Expenditure: federal/provincial; real exogenous except for military wage expenditure

Exports: demand; primarily price taker

Imports: demand; prices are mark-up on import costs

Prices: cost mark-up with capacity utilization term

Price Expectations: adaptive

Technology: Cobb-Douglas

Factors of Production: non-energy capital M&E; non-energy capital  
non-res; man-hours

Wages: extended Phillips curve

Exchange Rate: function of interest rate differentials, the  
merchandise trade balance as a percentage of GNP,  
long-term capital flows relative to GNP, and  
purchasing power parity; in the long-run gravitates  
to purchasing power parity

### **MACE**

Date of Model: December 1983

Use of Model: simulation and policy analysis, especially for  
examining linkages between energy and the economy

Number of Equations: 50 total; 25 stochastic in macro block;  
563 in energy block

Frequency of Data: Annual

Computer System: RDX2 Simulator and SHAZAM for estimation

End of Estimation Period: 1982

#### **Characterization of Sectors**

Consumption: life cycle model based on the after-tax wage and  
transfer income and the real value of household wealth

Investment in Residential

Construction: treated as part of non-energy capital investment

Investment: for non-energy sector is a function of gap between  
desired and actual capital stock based on technology  
and productivity costs relative to output price

Government Expenditure: no federal/provincial split; exogenous in  
real terms

Exports: both demand (world income & relative price) and supply  
(inventory disequilibrium); price based on foreign prices  
and domestic costs

Imports: demand; price taker



Prices: cost mark-up extended to include difference between actual and desired inventory stocks

Price Expectations: adaptive; not explicitly modelled

Technology: vintage CES bundling of energy and capital nested within a Cobb-Douglas function for gross output based on the bundle and efficiency units of labour

Factors of Production: capital-plus-energy; efficiency units of labour

Wages: function of percentage change in labour productivity, the price of absorption, the terms of trade and gross output relative to supply; unemployment dropped from latest version of model

Exchange Rate: several alternative regimes; portfolio model with limited intervention allows for exchange rate smoothing capital flows; exchange rate clears current account and net capital inflows.

## **SAM**

Date of Model: April, 1983

Use of Model: small theoretically-based model intended for medium- to long-term simulation analysis

Number of Equations: 129 total; 21 stochastic

Frequency of Data: Annual

Computer System: TSP; TROLL and other packages sometimes utilized for estimation

End of Estimation Period: mostly 1981

### **Characterization of Sectors**

Consumption: human and non-human wealth

Investment in Residential

Construction: aggregated with other industries

Investment: Jorgenson neoclassical model

Government

Expenditure: total government; non-wage endogenous in nominal terms based on exogenous share of domestic output and wage

determined by employment equation and wage rate.

Exports: hybrid incorporating both demand and supply; both price taker and setter

Imports: demand; price taker

Prices: novel formulation as a function of price level disequilibrium, excess demand, and underlying trend factors relating to money supply growth

Price Expectations: converges adaptively to the equilibrium rate of inflation determined by the money growth rate less the real growth rate

Technology: CES

Factors of Production: capital and energy bundled, and labour

Wages: long-run real wage in efficiency units depends only on technology and other steady state factor prices; in shorter-term exhibits Phillips curve-like behaviour

Exchange Rate: Moves adaptively to long-run equilibrium determined mainly by the interaction of supply and demand in the market for net foreign assets and by an interest rate parity condition

**APPENDIX 3**  
**STATE OF THE ART IN CANADIAN MACROECONOMETRIC MODELS**  
**LIST OF CONTACTS**

1. CANDIDE MODEL 2.0

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2. CHASE

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6. MTFM

Ernie Stokes	746-1261
Anselm London	746-1261

7. RDXF

Claude Simard	563-8378
Bruce Montador	563-8597

8. QFS

Don Drummond	993-6212
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Heather Robertson 993-6211

9. SAM

David Rose 563-8728

Jack Selody 563-8351

10. TIM

Mike McCracken 238-4831

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**APPENDIX 5**  
**PROCEEDINGS OF A CONFERENCE ON CANADIAN MACROECONOMIC**  
**MODELS**  
**SPONSORED BY THE ECONOMIC FORECASTING DIVISION,**  
**DEPARTMENT OF FINANCE**  
**IN OTTAWA ON NOVEMBER 30, 1984**

**INTRODUCTION**

In reviewing the first draft of The State of the Art in Canadian Macroeconomic Modelling, it became evident to officials in the Department of Finance that a modelling conference would be useful at this time. The momentum of modelling-type research projects had picked up quite considerably over the past year. A number of modelling groups were in the midst of major research projects, or had virtually completed major model revisions. However, written documentation of these efforts was not available when the status report was being prepared. Moreover, the documentation for existing models had not been completed by all groups, making it difficult, at times, to ascertain the underlying structure of some of the models.

Another general point of interest was that some modelling experts remarked on the complexity as well as the cost of "repairs and maintenance" of larger macroeconomic models, and were interested in building scaled-down versions of their existing models, or in reorganizing their model structure into "satellites".

There was, as well, a fair degree of uncertainty as to the impact of recent developments in electronic data processing on model research. Finally, all groups expressed interest in exchanging information relating to current developments in Canadian macroeconomic modelling. The agenda for the conference was designed to reflect these concerns as well as an interest at the Department of Finance in sharpening the focus of future modelling research for its own model, QFS.

**AGENDA**

There were five sessions at the conference. Each sought to answer a number of questions under the direction of a discussion leader.

### **RETROSPECTIVE AND PROSPECTIVE VIEW ON ECONOMETRIC MODELS.**

Discussion Leader: John Helliwell,  
University of British Columbia.)

### **ECONOMIC THEORY AND ITS APPLICATION TO ECONOMETRIC MODELLING.**

\*How can theory provide restrictions in estimation? When should these restrictions be applied? What is the role of data in validating theoretical priors?

Discussion Leader: David Rose,  
Bank of Canada.

### **OPTIMAL SIZE OF MACROECONOMIC MODELS.**

\*What determines the limitations on the size of macroeconomic models? How is a model's use or application reflected in its size and structure? How can satellites or submodels be constructed to vary the size and level of detail of a model according to its use?

Discussion Leader: Mike McCracken,  
Informetrica Ltd.

### **ONGOING MODELLING RESEARCH.**

\*What are each group's current modelling plans or recently completed research projects relating to theory, model structure, model organization, and documentation? Where is the broad thrust of model development headed? How does model use determine research projects? Are model "repairs and maintenance" a major part of ongoing research? Have recent developments in electronic data processing influenced the direction of future modelling research?

Discussion Leader: Patrick Grady,  
Grady Economics & Associates Ltd.

### **ESTABLISHING NETWORKS FOR FUTURE MODELLING RESEARCH.**

\*What are the best ways to develop and maintain contacts and to improve the flow of information among modellers?

Discussion Leader: Heather Robertson,  
Department of Finance.

### **Opening Remarks**

Scott Clark, General Director of the Fiscal Policy and Economic Analysis Branch of the Department of Finance, opened the conference by thanking those present for participating in the conference on modelling. Clark also expressed the Department's particular appreciation of the co-operation that was extended by all of the Canadian modelling groups during preparation of the report on The State of the Art in Canadian Macroeconomic Modelling. Clark described the conference as another step in the continuing co-operation of Canadian modellers. A clearer idea of how the co-operative effort can best continue in the future was expected to be one of the benefits of the day.

### **RETROSPECTIVE AND PROSPECTIVE VIEW ON ECONOMETRIC MODELS**

John Helliwell, University of British Columbia.

More than in any other country model building has been an open collaborative effort in Canada. A general understanding of the various models has consequently developed.

The process has been open from the beginning. While the origin of model-building in Canada was official, that is within government departments and agencies, the work was documented and made publically available. The large required investments such as for data bases were undertaken by government departments and made available to all as a public good..

The collaboration has been a very positive factor. It has prevented modellers from assuming exaggerated positions, but not at the cost of stifling a rich diversity among models. The diversity among Canadian models, in terms of disaggregation and theoretical structures, is about the same as in other countries where there has been less of a collaborative effort. Another aspect of the collaborative effort in Canada is that many of the people now having principal responsibility for a model were at one time assistants in one of the earlier efforts. Many people currently involved have been associated with more than one modelling project.

All modellers should continue this process of openness in allowing others access to the details of their model and in providing complete documentation. This is not only necessary for the modelling community to advance, but should also be beneficial for the individual organizations.

Advances in computer software and hardware have made models accessible to a larger number of people. Some onus is on modellers to provide good documentation so that proper use of their models can be made. When mapping out model development strategies an important consideration should be the accessibility of the model to outsiders.

With proper documentation modellers can empirically test each others' equations and theoretical structures. This will permit a better understanding.

Individual modelling gains will be made by ensuring that the "modelling community" operates openly, consistently, and scientifically.

Consistency in models is important. When imposed, theoretical properties must be consistent throughout a model and should be imposed at the maximum degree of transparency to users of the model. The "black box" phenomenon should be avoided, or at the very least, not encouraged. Logical structures should be kept trim. Models should be transformable into reduced forms.

Mike McCracken, Informetrica.

We should not make our models simple just to accomodate "intellectually lazy" people. To improve the state of models they will probably have to get more sophisticated. It is also not apparent that a small size is inherently desirable.

Modellers should not be overly concerned to downsize and simplify their models so that they can be accomodated by micro-computers. The capabilities of computers restricted the development and uses of models in the 1960s. A shift to modelling on micro-computers would just return all of those restrictions. However, micro-computer versions may be useful in giving a larger number of people an understanding of modelling and macroeconomics.

Helliwell's call for openness in the modelling process deserves support. All modellers have benefitted from the openness and have a public responsibility to contribute from their own work.

David Rose, Bank of Canada

The state of documentation can be as important as the size of a model and its inherent complexity. A model may be complex, but it should be explained simply and clearly.

It is important not to lose sight of macroeconomic considerations and consistency when

disaggregating models and making them more complex.

Claude Simard, Bank of Canada.

Helliwell noted the extensive efforts that went into the initial preparation of a database for models. Unfortunately, the "common database" is slipping in quality. In particular, the degrees of freedom are being reduced because when series are altered consistent historical data are often not created. Consequently, at present there is nothing like a 30-year base of historic data.

Mike McCracken, Informetrica.

When the initial databases were constructed they were built by modellers for modellers. This focus has been lost. Since 1969-70 when CANSIM was introduced, the database has become a warehouse of data. There has not been an attempt to provide consistency over time.

Krishna Sahay, Department of Finance.

Disaggregation and complexity often arise from the inability to empirically verify hypotheses at an aggregate level. This raises the issue of at what level we should impose constraints on models. Another point worth bearing in mind is that most models that have desirable theoretical responses do not forecast well.

Jean-Pierre Aubry, Bank of Canada.

Policy-makers had high expectations of models at the outset. It is clear that models have not fulfilled these expectations. We now have a sense of reality of our models. We know the limitations and the risks involved in their use. We should not try to hide these limitations.

Ron Bodkin, University of Ottawa.

David Rose is right in saying that clarifying models does not so much mean simplifying them as explaining them. Hence, models and the research going into their development should be documented in a public forum. This has not been the case in the past. It was very difficult obtaining information on the early Canadian modelling efforts for the purpose of writing a survey of Canadian

Ernie Stokes, Conference Board.

Account must be taken of the objective function of private firms offering model services. They usually can not afford to devote a lot of resources to model documentation.

Private companies are being forced to respond to a demand for small models available on microcomputers. There is pressure to follow the possible success of the Ray Fair model in the U.S..

Tom McCormack, Data Resources Canada.

The search for a good macroeconomic framework might involve a temporary move to aggregation in order to disaggregate from a consistent core.

Leo de Bever, Chase Econometrics.

Any failure of models is a failure of the model builders. Modellers have not responded quickly enough to developments. A good example is the energy shocks. Perhaps we have paid too much attention was paid to statistical results and not enough to the sense of equations. Many equations in our models produce nonsensical results. Why do we still put them in? In particular we accept equations that fit history even if we know they are unlikely to work in the future.

Ernie Stokes, Conference Board.

Where do we start to construct a model's foundation? What theory should we use?

Ross Preston, Economic Council.

Small models for microcomputers will not contribute much to policy analysis. They will help us to understand models and their use by emphasizing the rather primitive structures of large models. The movement to small models will be positive in that it will help the present non-user to accept larger, more complex models.

John Helliwell, University of British Columbia.

Making a model comprehensible does not mean that it must be simple enough to run on a microcomputer. Best-practice modelling is not necessarily highly aggregated. But the disaggregation must be consistent. The process mentioned by McCormack of aggregating and then disaggregating again is not necessary.

On the subject of theories in models a good test is for modellers to write down the ten best theories of macroeconomics and to see how they can fit into their model. Eight will vanish because they are non-operational or operationally equivalent to what is already in the model. This shows the extent to which much theoretical work has been non-operational.

Paul Jacobson, Informetrica.



Good documentation of a model can also be internally useful to an organization for training staff.

## **ECONOMIC THEORY AND ITS APPLICATION TO ECONOMETRIC MODELLING**

David Rose, Bank of Canada.

What is macroeconomics? Is it just an aggregation of micro relationships? Individual decision making may be the way the system works, but we are miles from that. There is no rigorous theory because of the difficulty of aggregating from micro to macro and the stringency of the required conditions. A strong adherence to a micro optimization and rational expectation approach leads to a conglomeration that is operationally useless. In market theory there is no macroeconomics. Macro theory has focussed on why markets fail. There is no good theory to link these two approaches. We have a business cycle theory. We have formal disequilibrium models but they are really non-operational because they do not explain why failures exist.

Critiques of models have come from Lucas and Sims' time series work. VARMA models are not the answer because they embody no real macro theory and tend to be every bit as unstable as structural models. They may be useful for forecasting, however.

The restrictions we put on Keynesian-type models do not have any status because they lack a formal rationale.

What does theory tell us to put into models? In a specific, practical sense very little. The crucial point is the interrelatedness of decision-making. But in terms of details there are many different things that could be put into a utility function. Some restrictions come from the fact that decisions are joint.

We must think carefully about whether to match data to theory or theory to data. In the SAM model there was a desire to have a simple (linear) consumption function but in order to do this wealth had to be defined in a particular way. The value of leisure had to be added. This is one case where theory suggested something specific.

Most theory is based on equilibrium and says little about disequilibrium or steady-state. This is not as rigid in dynamic disequilibrium.

Models often even fail to adequately incorporate the accounting system and stock-flow relationships. The end result can be a structure dependent on nothing more than exogenous trends and balancing items. This may not be too serious for short-run shock-minus-control simulations but these influences can seriously distort long-run properties.

Simultaneous estimation techniques are useful because they are more rigorous and highlight global properties. But it does mean that theoretical failures can get transferred to other sectors. Cross-equation restrictions can not be imposed using single-equation estimation techniques.

An important contribution of macroeconomics is that it organizes our thoughts into a consistent accounting framework. Often the accounting relationships and identities can be very important in determining dynamic behaviour.

Ernie Stokes, Conference Board.

There are two questions. What is the appropriate theory and how do we incorporate that theory. If the theory is wrong does consistency help?

John Helliwell, University of British Columbia.

We have more data points now so we can get better estimates of the adjustment process. Relative to David Rose, I would put more emphasis on dynamic adjustments.

It is often possible to get both better theoretical consistency and a better fit of the data.

When imposing constraints it is important to:

- test them against other theories;
- test them against the data;
- bring other aspects of the model into consistency.

David Rose, Bank of Canada.

We can not rely on estimation to determine the long-term equilibrium properties of a model. This must, by and large, be imposed. However, we can utilize estimation to learn more about the nature of the dynamic adjustments. This may also provide some useful information on long-run properties.

Ernie Stokes, Conference Board.

What is our reference point for long-run properties? Equilibrium in 5, 10, 15 years? Should the

model settle down quickly to shocks or take longer and go through cycles? MTFM takes longer to settle down and cycles more. SAM apparently settles down quickly.

David Rose, Bank of Canada.

Questions of when the economy will settle down into equilibrium are hypothetical. There is a group of people who believe the world is characterized by long-term disequilibrium.

Jean-Pierre Aubry, Bank of Canada.

Macroeconomic models give a narrow view for a few aggregates. All sectors of the economy do not move together. The models do not address distributions and sectoral balance.

Kevin Lynch, Department of Finance.

Do we have the right real-financial and real-fiscal linkages in models? Perhaps there have been problems in estimating the linkages because there have been small variances in the historical period but large variances now.

Mike McCracken, Informetrica.

At Informetrica we have looked for the influence of things like Debt/GNP but we have not found the empirical link. There is a danger in just responding to the popular wisdom of the moment by incorporating links which we can not verify.

John Helliwell, University of British Columbia.

Are links with debt necessary to explain high real interest rates? While the debt to GNP ratio only recently began to rise, it dropped significantly for most of the post-war period until 1975. It might be possible to get a significant relationship between real interest rates and the ratio of debt to GNP.

## OPTIMAL SIZE OF MACROECONOMIC MODELS

Mike McCracken, Informetrica.

In the early days of models the debate over small versus large revolved around 10 versus 100 equations. Now the debate is 100 versus 5000 equations.

The equation count is not the only measure of size. There is also the degree of simultaneity and the regional/sectoral dimension which must be considered.

It is not a question of the optimal size but the optimal size for the purpose of the model.

Why choose a larger model?

- detailed information is of value.
- increased accuracy for major aggregates.
- simpler specifications are possible.
- ability to detect structural change or data changes.
- more precise policy hooks.
- more precise simulations of impacts.

Does disaggregation improve the accuracy of the aggregates? Some U.S. tests suggest this is true.

Why choose a smaller model?

- pedagogical utility.
- ease of maintenance.

- reduced solution time.

- focus more effort on each equation.

Balance is important in a model. There should be comparable levels of disaggregation for different concepts. As an example, it is not very useful to have 50 price equations that are all driven off a single wage equation.

Aggregation will generate more complicated lag structures. For example, two equations each with dependent variables lagged one period give equations with lags of two periods when aggregated.

Claude Simard, Bank of Canada.

It is true that the advantage of disaggregation is that it gives a cleaner adjustment process. But the more disaggregated the model the more difficult it is to impose a macroeconomic structure.

The question of optimal model size is also related to the time period of a forecast. It is possible that large models are more accurate in the short-term, but that a small one might be better for the long-term. In the short-term, exogenous variables are more important in determining the growth path. In the longer-run, simultaneities become more important.

David Rose, Bank of Canada.

There are some restrictions that it is desirable to impose at a global level. The detail is necessary to determine relative prices which are in turn important for the allocation of output. But if there is no interest in the allocation, then the detail is unnecessary.

Ross Preston, Economic Council.

It is not appropriate to aggregate incongruent technologies on the production side. How is it possible to aggregate goods with constant returns, increasing returns, etc.?

John Helliwell, University of British Columbia.

It is not simply a matter of the production functions being different. For disaggregation it is necessary to explain the technology in each industry as well as inter-industry flows.

The choice on the level of disaggregation depends on the clients, their interests, and available resources. In Finance's case there is a need to move quickly. Did big models handle the energy shocks better? Probably not because energy was treated just like any other industry in these models. When something really big happens, it is usually necessary to rebuild a model.

Finance should start with the minimum required detail except for the government tax and expenditure sector where a lot of detail is necessary. It is probably best not to use the detail by sector and region to a great extent.

When the model gets too big control is lost. It is best to start with a minimum level of disaggregation. Results will be more forthcoming.

Ross Preston, Economic Council.

The solution to the problem is not to build small models so we can stay within tight budgets. The impacts of the analyses may be so important that the best solution is to increase the budgets devoted to modelling until large models can be run properly. In the past insufficient resources have been devoted to modelling.

Kevin Lynch, Department of Finance.

Can theoretical consistency be maintained throughout a 3,000-5,000 equation model? Also, are there not serious problems of a "hands-on" nature when doing forecasts and policy analysis with thousands of equations?

Mike McCracken, Informetrica.

With a disaggregated model it is easier to utilize outside experts in preparing a forecast and to have their advice explicitly incorporated in the model and forecast. It is hard to relate expertise to a global variable. This problem could also be addressed, however, through the use of satellites to a smaller model.

A danger of disaggregation, which must be borne in mind, is that sometimes the data are not accurate because there are mis-classifications between categories.

David Rose, Bank of Canada.

Size and detail depends on the purpose. For industrial strategy disaggregation may be desirable. For big issues it is more advantageous to keep the model small.

## **ONGOING MODELLING RESEARCH**

Patrick Grady, Grady Economics & Associates Ltd.

The time is never right to study the "state of the art" because models are always in a state of flux. In particular, the Economic Council has virtually finished a very large modelling project but the results are unfortunately not available for inclusion in the report. The FOCUS group also plans to do major redevelopment.

Each group is asked to speak on:

- the current status of model development and plans; and
- the impact of data processing developments on their plans.

### **CANDIDE**

Ross Preston, Economic Council

A year and a half ago a decision was made to rebuild CANDIDE 2.0. There was three months of debate on whether the size of the model should be reduced. It was finally decided to use the 45-sector level of detail.

A main weakness of CANDIDE 2.0 had been the consumer sector. Now in CANDIDE 3.0 stochastic equations are utilized for consumption rather than modelling the savings rate. Also, the theoretical derivations of the user cost terms in investment have been expanded.

The I-O sector is similar to the earlier version of the model but a big difference is that the new model does not have the error adjustment equations which sometimes dramatically affected the model dynamics of the earlier model.

The portfolio (excluding money) sector of the model was redone and a stand-alone money demand equation was added.

A list of the changes will be provided to Finance.

The multiplier in the new model is lower than in the previous version. After just topping 1, it goes back toward zero.

Many priors had to be imposed on the model to get what was deemed to be desired properties.

The estimation period for the model runs from the mid-50s to 1981 in general.

Another key feature of the new model is that it exhibits a much stronger price response to shocks than before.

Expectations terms were used in several sectors -- wages, savings, investment, exchange rate. The same specification is utilized in each place. A concern is that the model may now be overly sensitive to expectations. This is a good example of how the imposition of priors can effect the properties of the model.

The intention at the moment is to use CANDIDE 3.0 for the next annual review.

## **TIM**

Elizabeth Ruddick, Informetrica.

No major modelling projects are underway now, but there are a number of areas and issues that are under review. These include:

- The government sector reaction functions.
- The income and price elasticities of the trade sector.
- Further disaggregation of RDP (government and services).
- The distribution of pensions.
- The development of separate persons and unincorporated business sectors.
- The disaggregation of the participation rate for prime age males into a younger and an older group.
- The energy sector in light of the upcoming energy policy changes.
- Capital flows, interest rates, and the exchange rate are of primary concern.
- The provincial model and regional-sectoral detail have been the



focus of much effort.

## **CHASE**

Leo de Bever, Chase Econometrics.

No major modelling research is currently underway. Model development is in response to specific client requests. The work at the moment is more of the repair and maintenance type.

From the point of view of model management, it would be desirable to move to greater use of satellite models.

CHASE has developed econometric software for use on a microcomputer.

## **DRI**

Tom McCormack, Data Resources Canada.

There was an extensive round of model improvements in 1982. These were ready in early 1983. In 1983 the focus was on the financial sector and efforts to improve the documentation. So far in 1984 there has been no development work.

There are plans to offer a small model for use on a microcomputer which utilizes software developed by the U.S. DRI. This model, which was constructed in 1984 and has 100 to 125 equations, will be available sometime in 1985. The impetus for this model has come from clients who do not want the detail of the big model for all purposes. The small model has been designed to be very similar to the large DRI model in terms of dynamic properties.

## **FOCUS**

Andre Plourde, University of Toronto.

Three modelling projects are either underway, about to start, or are in the planning phase:

Ongoing work on FOCUS including:

- Re-estimation.

- A reworking of the housing sector.
- An effort to improve the long-run properties.
- Further work on the investment sector.

The development of a disaggregated supply-side model to capture key structural behavioural differences to be used for short-run simulations and perhaps to be later integrated with FOCUS.

Eventually the development of a more aggregated general equilibrium model.

## **MTFM**

Ernie Stokes, Conference Board.

There are several developments of interest.

- The number of output categories has been increased.
- A different user cost term has been incorporated in the investment sector (based on recent published work by Boadway, Bruce, and Mintz).
- The definition of permanent income has been modified to allow for the fact that the full value of government bonds should not be counted as net wealth due to the associated future tax liability.
- The wage equations have been re-estimated.

Work is underway on the documentation, but the timing of its release has not yet been decided.

Considerable work was done on the provincial model to generate provincial value-added by industry and to endogenize many of the exogenous variables.

A microcomputer version of TROLL is planned. It will not likely be adequate to simulate large models.

## **QFS**

Heather Robertson, Department of Finance.

The Department of Finance has a medium-sized model called QFS that in most respects is satisfactory but as with other models needs improvements in many areas. Part of the need for

improvement is that the use of QFS has shifted. Initially it was designed primarily for short-term forecasting, but now it is used to do medium-term forecasting and policy analysis. There is now consequently greater emphasis on the model's dynamic responses.

The Department intends to use the discussions of today's conference and The State of the Art in Canadian Macroeconomic Modelling report to help focus the development strategy for QFS improvement. In general we want to trim the size of the model, and use satellites where need be, improve the medium-term properties, but to retain the high degree of government sector detail.

There are several steps to the plans.

- Defining what other groups have done in order to focus our efforts and resources.
- Mapping out a skeleton version of what we think QFS should look like when completely revised.
- Establishing a block-structure research agenda by sector and/or concept.
- Recently or currently, changing the wage equation, income sector, price-tax linkages and labour supply.
- Investing a lot of effort in repairs and maintenance. In particular the documentation of the model and its properties and software for table creations etc. have been improved.

## **RDXF**

Claude Simard, Bank of Canada.

Some work is underway on the natural rate of unemployment, the cost of capital and incorporating the new trade data.

The major project has been the development of a supply block. This is designed to remedy problems of long-run consistency including inconsistencies between prices and the production function. The new approach to the supply-side introduces an interaction between price determination and investment.

Gross private business product, the key output concept in the model, has been split into three industries -- energy, raw materials, and a residual commercial. The production function utilized is translog. The block will be run parallel with the rest of the model until fully tested.

## **MACE**

John Helliwell, University of British Columbia.

MACE is a supply-driven model which was constructed mainly to provide a framework for testing alternative theories. The next phase of its development might be to run the model without its large energy satellite. Spencer Star at Laval has already done this in order to put MACE on a microcomputer.

Work is also being done on estimating MACE-type supply sectors for the 7 OECD major countries.

## **SAM**

Jack Selody, Bank of Canada.

The development work on SAM is complete for the present and the effort is shifting to documentation.

Possible future developments include:

- inclusion of the government stock of capital.
- division of public good in consumption.
- further work on demand for Canadian assets.
- work on the development of satellite models.

Some work has also been done to put SAM on a microcomputer. This only involved minor modifications. Unfortunately, the software which runs SAM is not user-friendly, thus further work is being done on the software to increase its user-friendliness.

## ESTABLISHING NETWORKS FOR FUTURE MODELLING RESEARCH

Heather Robertson, Department of Finance.

Heather Robertson informed participants that the proceedings of the conference will be appended to the report on The State of the Art in Canadian Macroeconomic Modelling. If desired, Finance will play the role of a clearing house for model research papers. The list in the draft sections of The State.... was given as an example of what could be provided. If people could send Finance copies of their modelling work, it could be added to a bibliography and circulated periodically. The frequency of distribution will depend on how many reports are received. People on the distribution list can contact the source if they wish a copy. There was a general interest expressed at the conference in having Finance play this role. Finance will also undertake periodically to update The State.... report if the other modelling groups will assist with their sections when major changes are made.

Heather Robertson also observed that there is a gap in the knowledge available of the dynamic properties and that The State.... report did not go into this. Helliwell suggested appending summary tables from the 1982 Comparative Models Seminar. Ross Preston said he would provide a dynamics paper in February or March with documentation on the model to follow. The paper will compare the responses with the previous version. Tom McCormack said he could provide tables of responses from a more recent version of DRI. Claude Simard said the same could be done for RDXF.

Heather Robertson turned to a discussion of data problems in common among modellers. Claude Simard repeated that we need better historical data on a consistent basis. It was noted that the problem will be worsened with the 1981 rebasing in that consistent historical data will not be made available. It was suggested by several participants that as modellers we should be concerned about this issue and emphasize the need for consistent series.

There was some discussion of a possible need for a new forum to address issues of data with Statistics Canada. Mike McCracken pointed out that there were already several Statistics Canada advisory committees and that previous efforts to establish a users group such as was suggested were not successful.

Don Drummond raised the issue whether other modelling groups used a lot of data from Statistics Canada that are not available on CANSIM. It was universally agreed that this is a major

problem

that delays the database updating process. John Helliwell suggested that Finance circulate to the model groups the list of variables in this category for Finance and that the model groups check the

variables in common and add any others. Finance agreed to do this quickly. The result would then be a joint request to Statistics Canada for CANSIM-availability of widely-used series.

## **Closing Comments**

David Dodge, the Assistant Deputy Minister of the Fiscal Policy and Economic Analysis Branch of the Department of Finance, closed the conference by stressing the utility of such gatherings to model builders. He suggested that the dialogue among modelling groups should be made more frequent and systematic, and that networks should be established. He expressed the Department of Finance's interest in continuing to maintain an up-to-date listing of research papers, model documentation and standard simulation results, and in providing a forum for modellers to present model proposals or recently completed modelling efforts.

Dodge also noted that the discussions at the conference covered topics of direct concern to the Department in its own modelling efforts. As such, it will provide useful input into the Department of Finance's model development plans.

Dodge summarized the role of the QFS model in the Canadian macroeconomic environment. QFS, like other Canadian models, has a multipurpose role. It is used to prepare quarterly national short- and medium-term economic projections, which are the key input into the preparation of the Fiscal Plan. The national forecast results are also the basis for the preparation of regional economic and provincial government forecasts. Furthermore, the model is utilized in the preparation of impact studies in analyzing the economic effects of various budget initiatives.

Because of this central role in the determination of fiscal policy projections and impact studies it is vital that QFS be "state-of-the-art" in terms of quality, consistency, and flexibility. The model should, in general, also be in the mainstream in terms of economic theory and dynamic responses.

This is one reason why it is important for the Department of Finance to maintain contacts with other modellers and not to embark, in isolation, on large scale development projects.

The current work on QFS involved a number of steps. A background piece on the current status of the QFS model, identifying the major problem areas was prepared. A series of criteria on which

to choose among types of models was laid out. In considering the underlying theory and structure of the model, the issues on the optimal size of the model, and the way in which economic theory should be applied were left unresolved. The discussions on these issues at the conference will be

useful for future modelling work.

The State of the Art in Canadian Macroeconomic Modelling prepared by Patrick Grady was another step in the research strategy. The purpose of this document was to describe the most recent modelling developments in order to assist in determining mainstream thoughts in Canadian economic models. It is, in a sense, the twin volume to the large comparative models binder

that was prepared two years ago. The State of the Art in Canadian Macroeconomic Modelling document summarizes the models' structure and underlying theory, while the comparative models binder summarizes the models' dynamics.

Each modelling group has received the parts of the document pertaining to its own model. As soon as the comments of the groups are received and incorporated, a revised draft of the whole document will be distributed to all participants in the conference. A summary proceedings of this conference will be added to the document as an appendix.

Dodge stressed that it is important to keep information on model structure up-to-date. Consequently, he said that with the continuing cooperation of modelling groups the Department of Finance will attempt to revise periodically The State.... paper as model changes are made. Also, if modelling experts send in summary tables of their model dynamics as suggested, then a complete library of Canadian models' structure and dynamics will be at hand and will be made available to all model participants.

Dodge concluded by thanking all of the participants at the conference on behalf of the Department of Finance.

**APPENDIX 6**  
**CONFERENCE ON CANADIAN MACROECONOMIC MODELLING**  
**ATTENDANTS**

**ROUND-TABLE DISCUSSANTS**

David Dodge	ADM, Finance
C. Scott Clark	General Director, Finance
Kevin Lynch	Director, Finance
Claude Simard	RDXF, Bank of Canada
David Rose	SAM, Bank of Canada
Jack Selody	SAM, Bank of Canada
Jean-Pierre Aubry	RDX2, Bank of Canada
Andre Plourde	FOCUS, University of Toronto
Mary MacGregor	MACE, University of Toronto
John Helliwell	MACE, University of British Columbia
Ernie Stokes	MTFM, Conference Board
Ross Preston	CANDIDE, Economic Council
Someshwar Rao	CANDIDE, Economic Council
Heather Robertson	QFS, Finance
Don Drummond	QFS, Finance
George Vasic	DRI, Data Resources Canada
Tom McCormack	DRI, Data Resources Canada
Mike McCracken	TIM, Informetrica Ltd.
Elizabeth Ruddick	TIM, Informetrica Ltd.
Leo de Bever	CHASE, Chase Econometrics Ltd.
Jan van Vliet	CHASE, Chase Econometrics Ltd.
Krishna Sahay	Chief, Finance
Patrick Grady	Grady Economics & Associates Ltd.

**GUEST OBSERVERS**



Paul Jacobson	Informetrica Ltd.
Hashem Hoda	Informetrica Ltd.
Ronald Bodkin	University of Ottawa
David Longworth	Bank of Canada
Pierre Duguay	Bank of Canada
Gordon Schaeffer	Bank of Canada
Patrice Muller	Bank of Canada
Peter Gusen	Conference Board

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John Hayward	Economic Forecasting Division
Pat Murray	Special Projects and Policy Analysis Division
Louis Grignon	Special Projects and Policy Analysis Division
John Lester	Fiscal Policy Division
Ed Kolano	Fiscal Policy Division
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Mike Kennedy	Monetary Analysis Division
Jon Cockerline	Monetary Analysis Division
Jim McCollum	Exchange Market Section

### **ECONOMIC PROGRAMS AND GOVERNMENT FINANCE BRANCH**

William Forward	Economic Development Division
Robert Rand	Economic Development Division

### **FEDERAL-PROVINCIAL RELATIONS AND SOCIAL POLICY BRANCH**

B. Nutting	Social Policy Division
Bob McLarty	Federal-Provincial Relations Division

### **TAX POLICY AND LEGISLATION BRANCH**

George Kuo	Commodity Taxes Section
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